CHAPTER 4

CHAPTER 4 Triangles and Congruence

Chapter Outline

In this chapter, you will learn all about triangles. First, we will learn about the properties of triangles and the angles within a triangle. Second, we will use that information to determine if two different triangles are congruent. After proving two triangles are congruent, we will use that information to prove other parts of the triangles are congruent as well as the properties of equilateral and isosceles triangles.

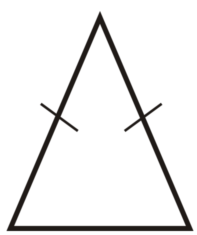
4.1 Triangle Sums

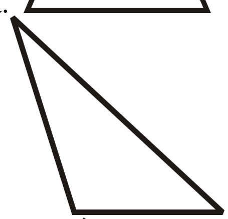
Learning Objectives

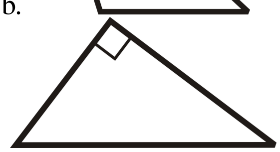
- Understand and apply the Triangle Sum Theorem.
- Identify interior and exterior angles in a triangle.
- Understand the Exterior Angle Theorem.

Review Queue

Classify the triangles below by their angles and sides.

a. 

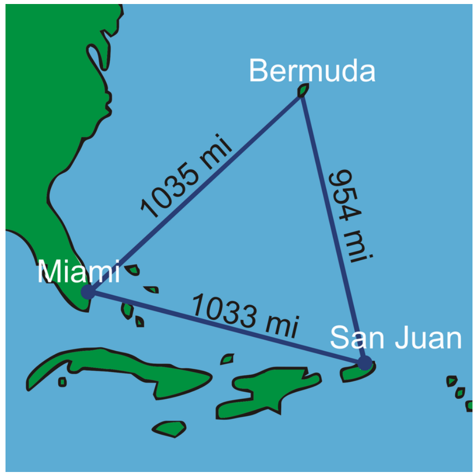
b. 

c. 

d. How many degrees are in a straight angle? Draw and label a straight angle, $\angle ABC$.

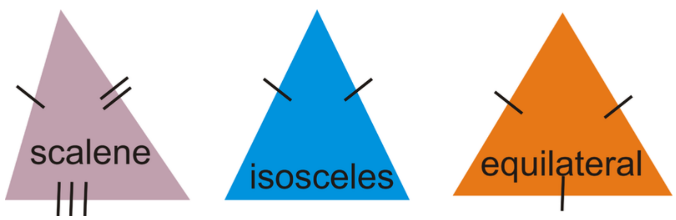
Know What? To the right is the Bermuda Triangle. You are probably familiar with the myth of this triangle; how several ships and planes passed through and mysteriously disappeared.

The measurements of the sides of the triangle are in the image. What type of triangle is this? Classify it by its sides and angles. Using a protractor, find the measure of each angle in the Bermuda Triangle. What do they add up to? Do you think the three angles in this image are the same as the three angles in the *actual* Bermuda triangle? Why or why not?



A Little Triangle Review

Recall that a triangle can be classified by its sides.



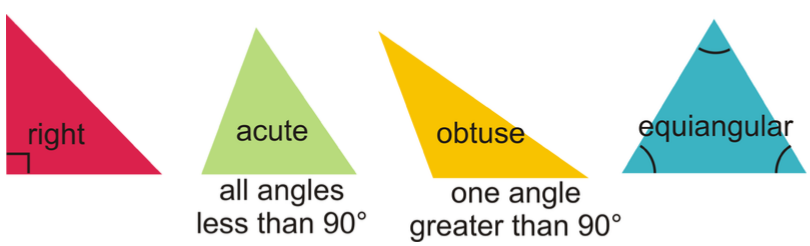
Scalene: All three sides are different lengths.

Isosceles: At least two sides are congruent.

Equilateral: All three sides are congruent.

By the definition, an equilateral triangle is also an isosceles triangle.

And, triangles can also be classified by their angles.



Right: One right angle.

Acute: All three angles are less than 90°.

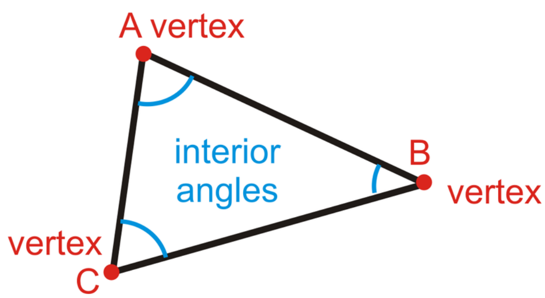
Obtuse: One angle is greater than 90°.

Equiangular: All three angles are congruent.

Triangle Sum Theorem

Interior Angles (in polygons): The angles inside of a closed figure with straight sides.

Vertex: The point where the sides of a polygon meet.



Triangles have three interior angles, three vertices and three sides.

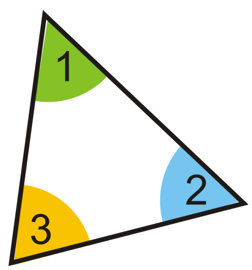
A triangle is labeled by its vertices with a $\triangle$**.** This triangle can be labeled $\triangle ABC, \triangle ACB, \triangle BCA, \triangle BAC, \triangle CBA$ or $\triangle CAB$. Order does not matter.

The angles in any polygon are measured in degrees. Each polygon has a different sum of degrees, depending on the number of angles in the polygon. How many degrees are in a triangle?

Investigation 4-1: Triangle Tear-Up

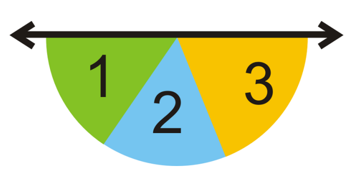
Tools Needed: paper, ruler, pencil, colored pencils

a. Draw a triangle on a piece of paper. Try to make all three angles different sizes. Color the three interior angles three different colors and label each one, $\angle 1, \angle 2$, and $\angle 3$.



b. Tear off the three colored angles, so you have three separate angles.

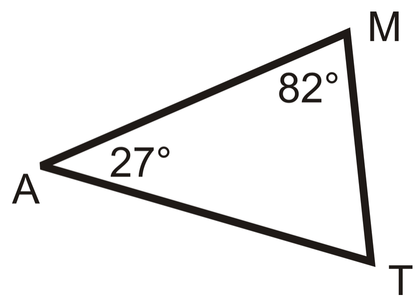
c. Attempt to line up the angles so their points all match up. What happens? What measure do the three angles add up to?



This investigation shows us that the sum of the angles in a triangle is 180° because the three angles fit together to form a straight line. Recall that a line is also a straight angle and all straight angles are 180°.

Triangle Sum Theorem: The interior angles of a triangle add up to 180°.

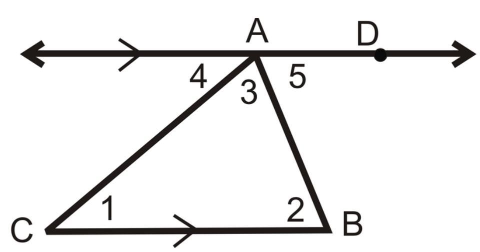
Example 1: What is the $m\angle T$?



Solution: From the Triangle Sum Theorem, we know that the three angles add up to 180°. Set up an equation to solve for $\angle T$.

$$\begin{aligned} m\angle M + m\angle A + m\angle T &= 180^\circ \\ 82^\circ + 27^\circ + m\angle T &= 180^\circ \\ 109^\circ + m\angle T &= 180^\circ \\ m\angle T &= 71^\circ \end{aligned}$$

Investigation 4-1 is one way to show that the angles in a triangle add up to 180°. However, it is not a two-column proof. Here we will prove the Triangle Sum Theorem.



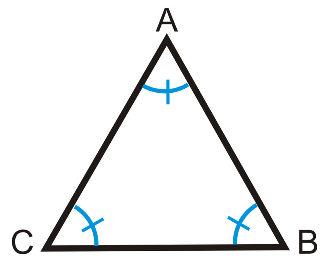
Given: $\triangle ABC$ with $\overleftrightarrow{AD} \parallel \overline{BC}$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$

TABLE 4.1:

Statement	***Reason***
1. $\triangle ABC$ above with $\overleftrightarrow{AD} \parallel \overline{BC}$	Given
2. $\angle 1 \cong \angle 4, \angle 2 \cong \angle 5$	Alternate Interior Angles Theorem
3. $m\angle 1 = m\angle 4, m\angle 2 = m\angle 5$	$\cong$ angles have = measures
4. $m\angle 4 + m\angle CAD = 180^\circ$	Linear Pair Postulate
5. $m\angle 3 + m\angle 5 = m\angle CAD$	Angle Addition Postulate
6. $m\angle 4 + m\angle 3 + m\angle 5 = 180^\circ$	Substitution PoE
7. $m\angle 1 + m\angle 3 + m\angle 2 = 180^\circ$	Substitution PoE

Example 2: What is the measure of each angle in an equiangular triangle?



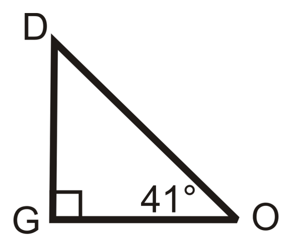
Solution: $\triangle ABC$ to the left is an example of an equiangular triangle, where all three angles are equal. Write an equation.

$$\begin{aligned} m\angle A + m\angle B + m\angle C &= 180^\circ \\ m\angle A + m\angle A + m\angle A &= 180^\circ \\ 3m\angle A &= 180^\circ \\ m\angle A &= 60^\circ \end{aligned}$$

If $m\angle A = 60^\circ$, then $m\angle B = 60^\circ$ and $m\angle C = 60^\circ$.

Theorem 4-1: Each angle in an equiangular triangle measures 60°.

Example 3: Find the measure of the missing angle.



Solution: $m\angle O = 41^\circ$ and $m\angle G = 90^\circ$ because it is a right angle.

$$\begin{aligned} m\angle D + m\angle O + m\angle G &= 180^\circ \\ m\angle D + 41^\circ + 90^\circ &= 180^\circ \\ m\angle D + 41^\circ &= 90^\circ \\ m\angle D &= 49^\circ \end{aligned}$$

Notice that $m\angle D + m\angle O = 90^\circ$ because $\angle G$ is a right angle.

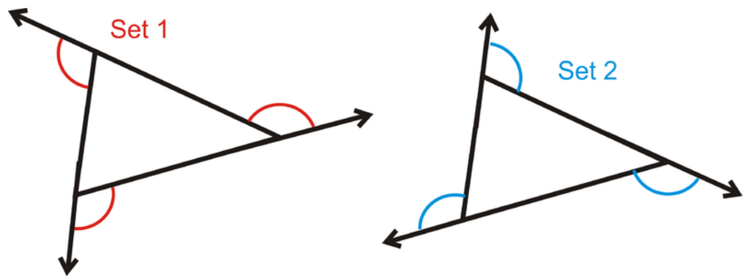
Theorem 4-2: The acute angles in a right triangle are always complementary.

Exterior Angles

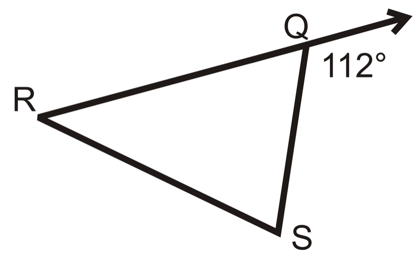
Exterior Angle: The angle formed by one side of a polygon and the extension of the adjacent side.

In all polygons, there are **two** sets of exterior angles, one going around the polygon clockwise and the other goes around the polygon counterclockwise.

By the definition, the interior angle and its adjacent exterior angle form a linear pair.



Example 4: Find the measure of $\angle RQS$.

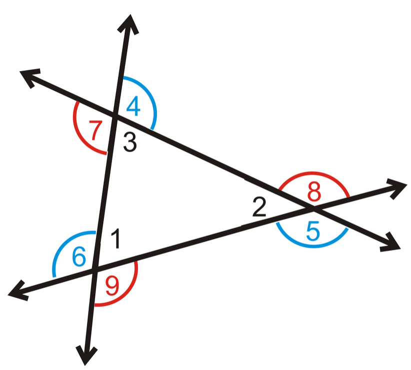


Solution: 112° is an exterior angle of $\triangle RQS$. Therefore, it is supplementary to $\angle RQS$ because they are a linear pair.

$$112^\circ + m\angle RQS = 180^\circ$$
$$m\angle RQS = 68^\circ$$

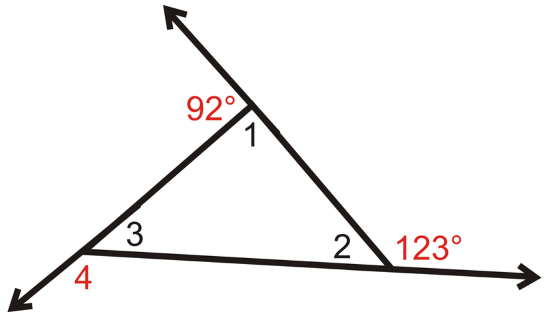
If we draw both sets of exterior angles on the same triangle, we have the following figure:

Notice, at each vertex, the exterior angles are also vertical angles, therefore they are congruent.



$$\angle 4 \cong \angle 7$$
$$\angle 5 \cong \angle 8$$
$$\angle 6 \cong \angle 9$$

Example 5: Find the measure of the numbered interior and exterior angles in the triangle.



Solution:

$m\angle 1 + 92^\circ = 180^\circ$ by the Linear Pair Postulate, so $m\angle 1 = 88^\circ$.

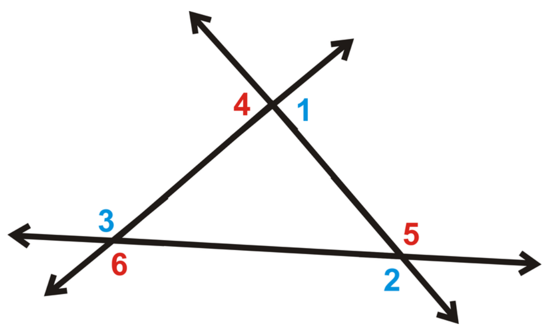
$m\angle 2 + 123^\circ = 180^\circ$ by the Linear Pair Postulate, so $m\angle 2 = 57^\circ$.

$m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$ by the Triangle Sum Theorem, so $88^\circ + 57^\circ + m\angle 3 = 180^\circ$ and $m\angle 3 = 35^\circ$.

$m\angle 3 + m\angle 4 = 180^\circ$ by the Linear Pair Postulate, so $m\angle 4 = 145^\circ$.

Looking at Example 5, the exterior angles are $92^\circ, 123^\circ$, and 145°. If we add these angles together, we get $92^\circ + 123^\circ + 145^\circ = 360^\circ$. This is always true for any set of exterior angles for any polygon.

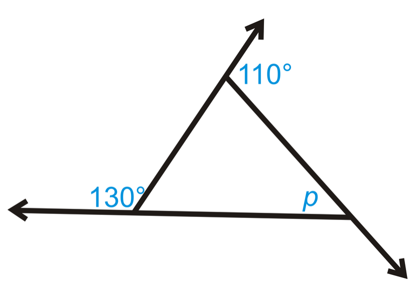
Exterior Angle Sum Theorem: Each set of exterior angles of a polygon add up to 360°.



$$m\angle 1 + m\angle 2 + m\angle 3 = 360^\circ$$
$$m\angle 4 + m\angle 5 + m\angle 6 = 360^\circ$$

We will prove this theorem for triangles in the review questions and will prove it for all polygons later in this text.

Example 6: What is the value of p in the triangle below?



Solution: First, we need to find the missing exterior angle, we will call it x. Set up an equation using the Exterior Angle Sum Theorem.

$$\begin{aligned}130^\circ + 110^\circ + x &= 360^\circ \\ x &= 360^\circ - 130^\circ - 110^\circ \\ x &= 120^\circ\end{aligned}$$

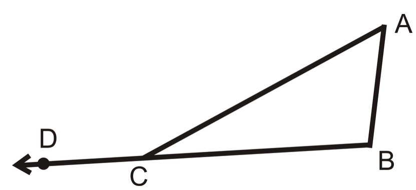
x and p are supplementary and add up to 180°.

$$\begin{aligned}x + p &= 180^\circ \\ 120^\circ + p &= 180^\circ \\ p &= 60^\circ\end{aligned}$$

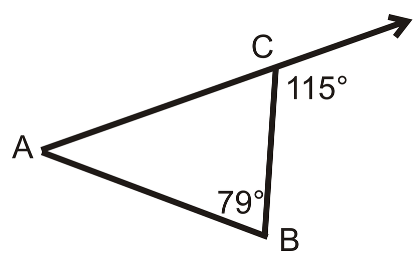
Exterior Angles Theorem

Remote Interior Angles: The two angles in a triangle that are not adjacent to the indicated exterior angle.

$\angle A$ and $\angle B$ are the remote interior angles for exterior angle $\angle ACD$.



Example 7: Find $m\angle A$.



Solution: First, find $m\angle ACB$. $m\angle ACB + 115^\circ = 180^\circ$ by the Linear Pair Postulate, so $m\angle ACB = 65^\circ$.

$m\angle A + 65^\circ + 79^\circ = 180^\circ$ by the Triangle Sum Theorem, so $m\angle A = 36^\circ$.

In Example 7, $m\angle A + m\angle B$ is $36^\circ + 79^\circ = 115^\circ$. This is the same as the exterior angle at C, 115°.

From this example, we can conclude the Exterior Angle Theorem.

Exterior Angle Theorem: The sum of the remote interior angles is equal to the non-adjacent exterior angle.

From the picture above, this means that $m\angle A + m\angle B = m\angle ACD$.

Here is the proof of the Exterior Angle Theorem. From the proof, you can see that this theorem is a combination of the Triangle Sum Theorem and the Linear Pair Postulate.

Given: $\triangle ABC$ with exterior angle $\angle ACD$

Prove: $m\angle A + m\angle B = m\angle ACD$

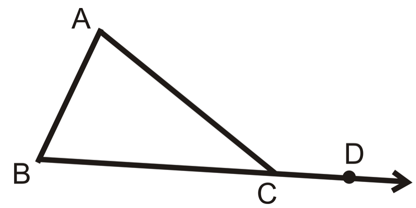
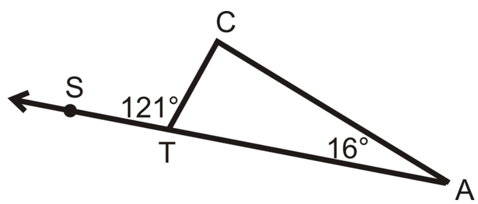


TABLE 4.2:

Statement	Reason
1. $\triangle ABC$ with exterior angle $\angle ACD$	Given
2. $m\angle A + m\angle B + m\angle ACB = 180^\circ$	Triangle Sum Theorem
3. $m\angle ACB + m\angle ACD = 180^\circ$	Linear Pair Postulate
4. $m\angle A + m\angle B + m\angle ACB = m\angle ACB + m\angle ACD$	Transitive PoE
5. $m\angle A + m\angle B = m\angle ACD$	Subtraction PoE

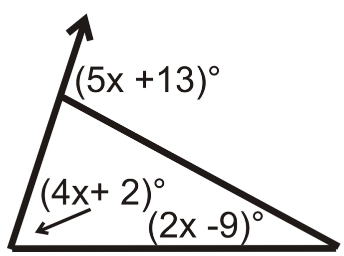
Example 8: Find $m\angle C$.



Solution: Using the Exterior Angle Theorem, $m\angle C + 16^\circ = 121^\circ$. Subtracting 16° from both sides, $m\angle C = 105^\circ$.

It is important to note that if you forget the Exterior Angle Theorem, you can do this problem just like we solved Example 7.

Example 9: ***Algebra Connection*** Find the value of x and the measure of each angle.



Solution: Set up an equation using the Exterior Angle Theorem.

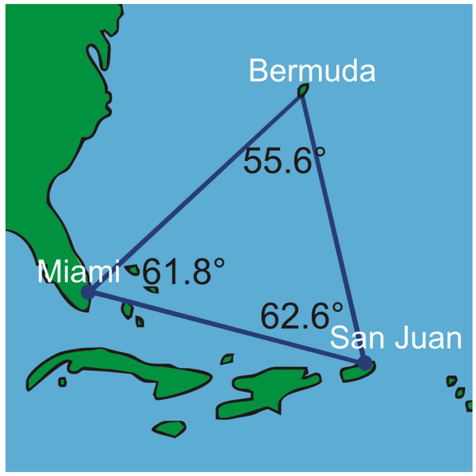
$$\underbrace{(4x+2)^\circ + (2x-9)^\circ}_{\text{interior angles}} = \underbrace{(5x+13)^\circ}_{\text{exterior angle}}$$

$$(6x-7)^\circ = (5x+13)^\circ$$
$$x = 20^\circ$$

Substituting 20° back in for x, the two interior angles are $(4(20)+2)^\circ = 82^\circ$ and $(2(20)-9)^\circ = 31^\circ$. The exterior angle is $(5(20)+13)^\circ = 113^\circ$. Double-checking our work, notice that $82^\circ + 31^\circ = 113^\circ$. If we had done the problem incorrectly, this check would not have worked.

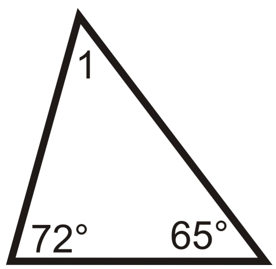
Know What? Revisited The Bermuda Triangle is an acute scalene triangle. The angle measures are in the picture to the right. Your measured angles should be within a degree or two of these measures. The angles should add up to 180°. However, because your measures are estimates using a protractor, they might not exactly add up.

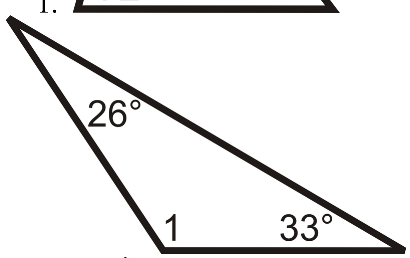
The angle measures in the picture are the actual measures, based off of the distances given, however, your measured angles might be off because the drawing is not to scale.



Review Questions

Determine $m\angle 1$.

1. 

2. 

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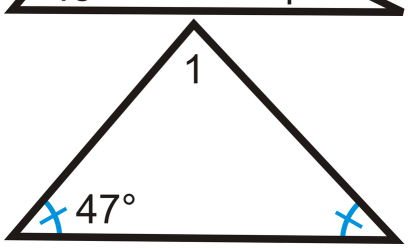
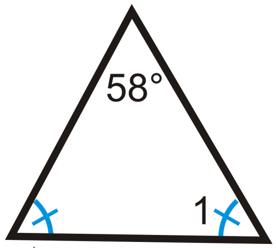
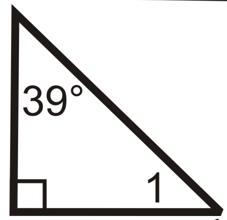
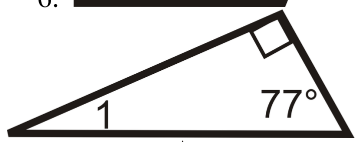
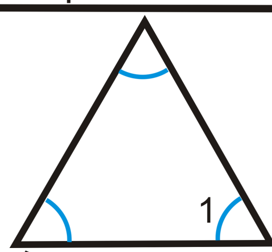
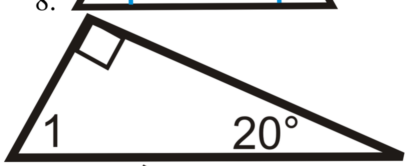
4. 

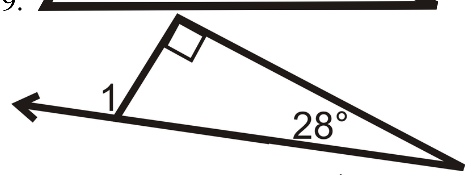
5. 

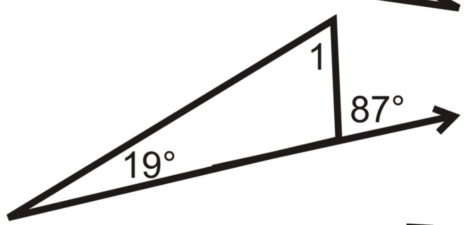
6. 

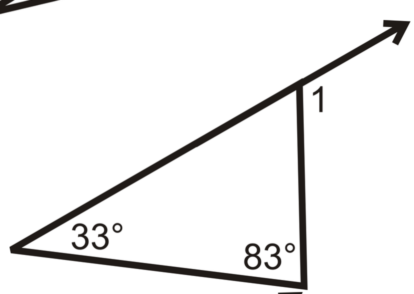
7. 

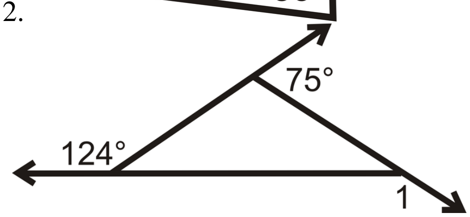
8. 

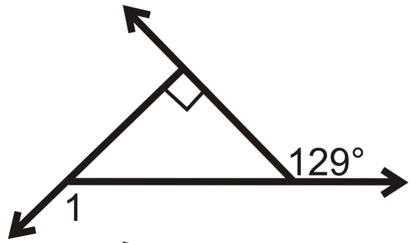
9. 

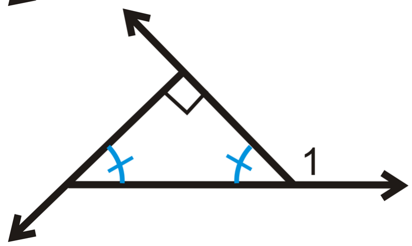
10. 

11. 

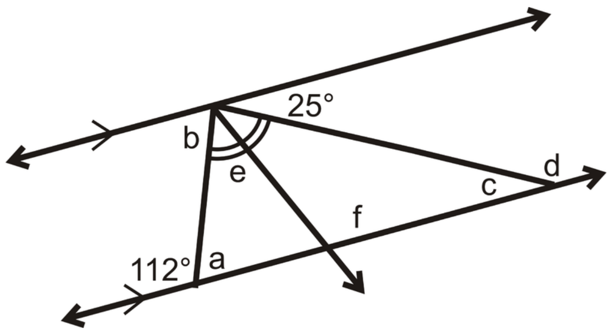
12. 

13. 

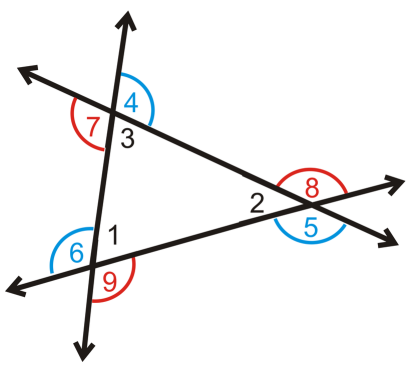
14. 

15. 

16. Find the lettered angles, $a - f$, in the picture to the right. Note that the two lines are parallel.



17. Fill in the blanks in the proof below. Given: The triangle to the right with interior angles and exterior angles. Prove: $m\angle 4 + m\angle 5 + m\angle 6 = 360^\circ$

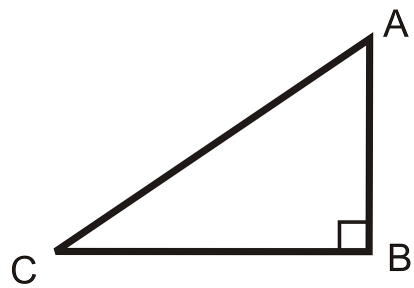


Only use the blue set of exterior angles for this proof.

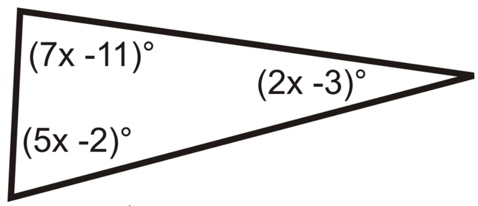
TABLE 4.3:

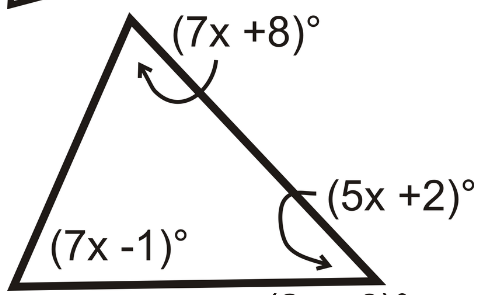
Statement	***Reason***
1. Triangle with interior and exterior angles.	Given
2. $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$	
3. $\angle 3$ and $\angle 4$ are a linear pair, $\angle 2$ and $\angle 5$ are a linear pair, and $\angle 1$ and $\angle 6$ are a linear pair	
4.	Linear Pair Postulate (do all 3)
5. $m\angle 1 + m\angle 6 = 180^\circ$ m $\angle 2 + m\angle 5 = 180^\circ$ m $\angle 3 + m\angle 4 = 180^\circ$	
6. $m\angle 1 + m\angle 6 + m\angle 2 + m\angle 5 + m\angle 3 + m\angle 4 = 540^\circ$	
7. $m\angle 4 + m\angle 5 + m\angle 6 = 360^\circ$	

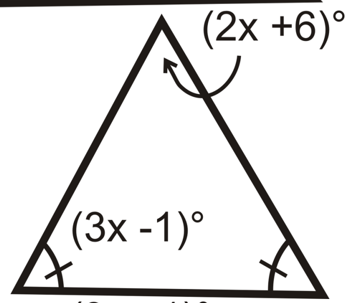
18. Write a two-column proof . <u>Given</u>: $\triangle ABC$ with right angle B. <u>Prove</u>: $\angle A$ and $\angle C$ are complementary.

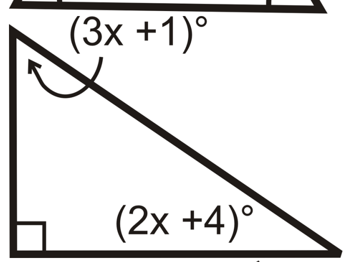


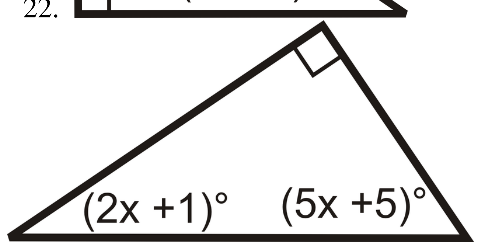
Algebra Connection Solve for x.

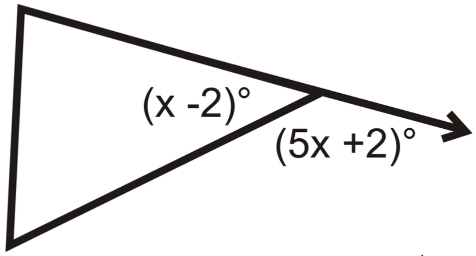
19. 

20. 

21. 

22. 

23. 



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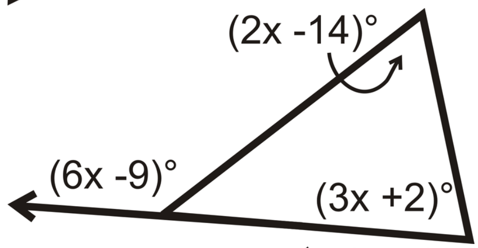

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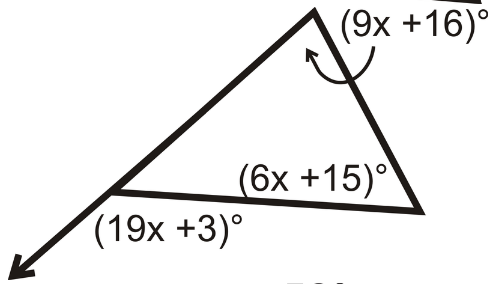

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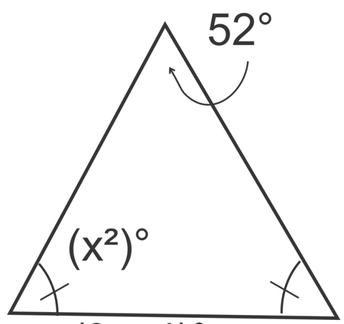

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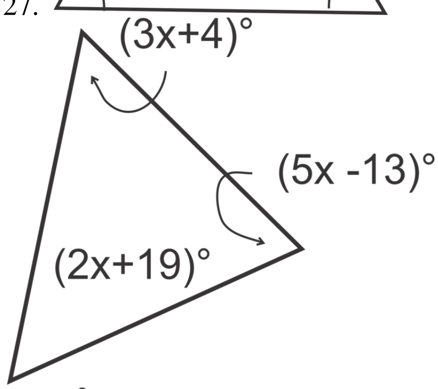

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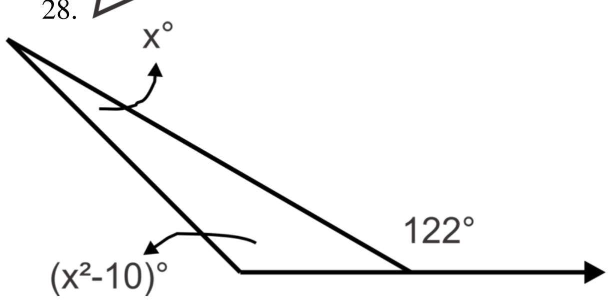

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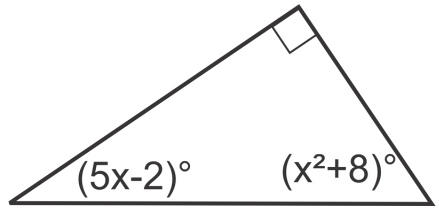
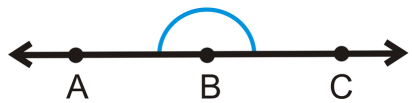
30. 

Review Queue Answers

a. acute isosceles
b. obtuse scalene
c. right scalene
d. 180°,



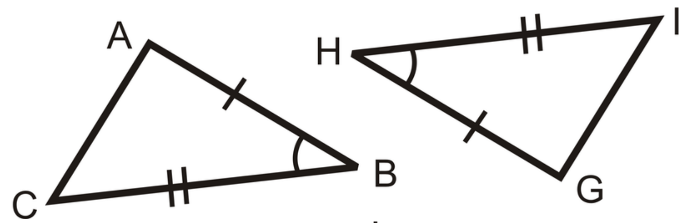
4.2 Congruent Figures

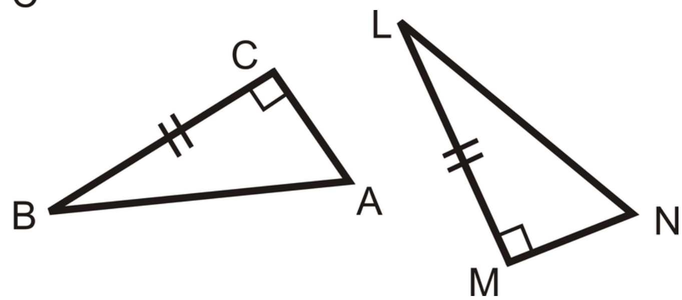
Learning Objectives

- Define congruent triangles and use congruence statements.
- Understand the Third Angle Theorem.
- Use properties of triangle congruence.

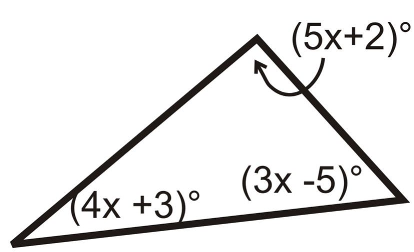
Review Queue

Which corresponding parts of each pair of triangles are congruent? Write all congruence statements for Questions 1 and 2.

a. 

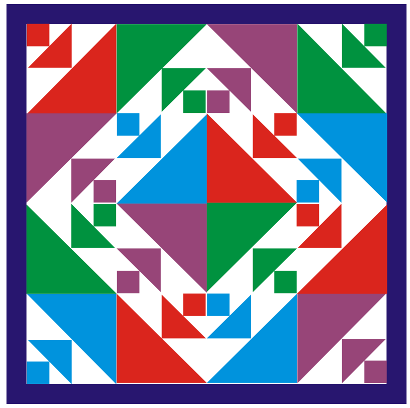
b. 

c. Determine the measure of x.



Know What? Quilt patterns are very geometrical. The pattern to the right is made up of several congruent figures. In order for these patterns to come together, the quilter rotates and flips each block (in this case, a large triangle, smaller triangle, and a smaller square) to get new patterns and arrangements.

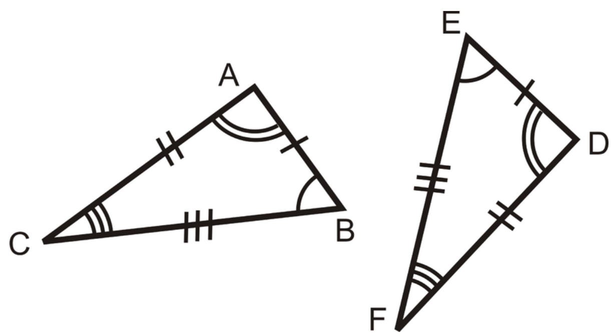
How many different sets of colored congruent triangles are there? How many triangles are in each set? How do you know these triangles are congruent?



Congruent Triangles

Recall that two figures are congruent if and only if they have exactly the same size and shape.

Congruent Triangles: Two triangles are congruent if the three corresponding angles and sides are congruent.

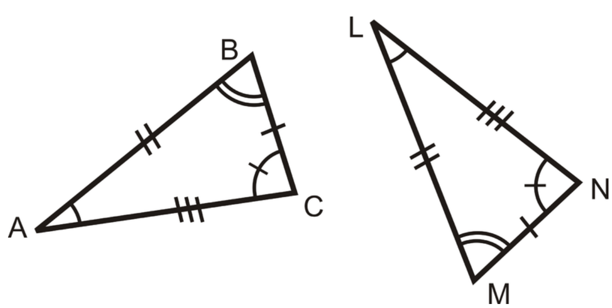


$\triangle ABC$ and $\triangle DEF$ are congruent because

$$\begin{array}{ccc} \overline{AB} \cong \overline{DE} & & \angle A \cong \angle D \\ \overline{BC} \cong \overline{EF} & \text{and} & \angle B \cong \angle E \\ \overline{AC} \cong \overline{DF} & & \angle C \cong \angle F \end{array}$$

When referring to corresponding congruent parts of triangles it is called **C**orresponding **P**arts of **C**ongruent **T**riangles are **C**ongruent, or **CPCTC**.

Example 1: Are the two triangles below congruent?



Solution: To determine if the triangles are congruent, each pair of corresponding sides and angles must be congruent.

Start with the sides and match up sides with the same number of tic marks. Using the tic marks: $\overline{BC} \cong \overline{MN}, \overline{AB} \cong \overline{LM}, \overline{AC} \cong \overline{LN}$.

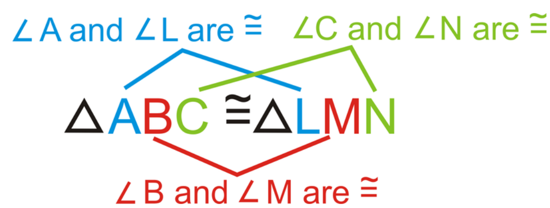
Next match the angles with the same markings; $\angle A \cong \angle L, \angle B \cong \angle M$, and $\angle C \cong \angle N$. Because all six parts are congruent, the two triangles are congruent.

We will learn, later in this chapter that it is impossible for two triangles to have all six parts be congruent and the triangles are not congruent, *when they are drawn to scale.*

Creating Congruence Statements

Looking at Example 1, we know that the two triangles are congruent because the three angles and three sides are congruent to the three angles and three sides in the other triangle.

When stating that two triangles are congruent, the order of the letters is very important. Corresponding parts must be written in the same order. Using Example 1, we would have:



Notice that the congruent sides also line up within the congruence statement.

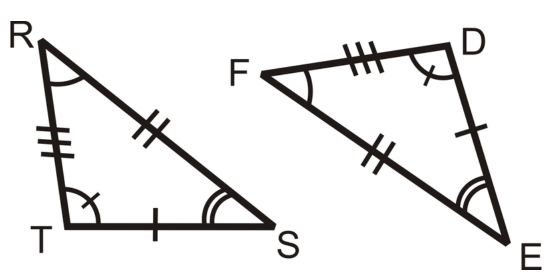
$$\overline{AB} \cong \overline{LM}, \overline{BC} \cong \overline{MN}, \overline{AC} \cong \overline{LN}$$

We can also write this congruence statement several other ways, as long as the congruent angles match up. For example, we can also write $\triangle ABC \cong \triangle LMN$ as:

$$\begin{array}{ll} \triangle ACB \cong \triangle LNM & \triangle BCA \cong \triangle MNL \\ \triangle BAC \cong \triangle MLN & \triangle CBA \cong \triangle NML \\ \triangle CAB \cong \triangle NLM & \end{array}$$

One congruence statement can always be written six ways. Any of the six ways above would be correct when stating that the two triangles in Example 1 are congruent.

Example 2: Write a congruence statement for the two triangles below.



Solution: To write the congruence statement, you need to line up the corresponding parts in the triangles: $\angle R \cong \angle F, \angle S \cong \angle E$, and $\angle T \cong \angle D$. Therefore, the triangles are $\triangle RST \cong \triangle FED$.

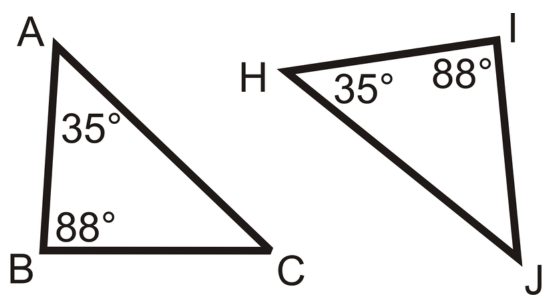
Example 3: If $\triangle CAT \cong \triangle DOG$, what else do you know?

Solution: From this congruence statement, we can conclude three pairs of angles and three pairs of sides are congruent.

$$\angle C \cong \angle D \qquad \angle A \cong \angle O \qquad \angle T \cong \angle G$$
$$\overline{CA} \cong \overline{DO} \qquad \overline{AT} \cong \overline{OG} \qquad \overline{CT} \cong \overline{DG}$$

The Third Angle Theorem

Example 4: Find $m\angle C$ and $m\angle J$.



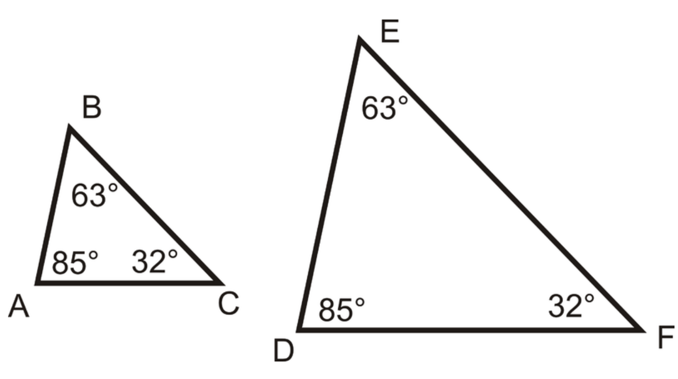
Solution: The sum of the angles in each triangle is 180°. So, for $\triangle ABC, 35^\circ + 88^\circ + m\angle C = 180^\circ$ and $m\angle C = 57^\circ$. For $\triangle HIJ$, $35^\circ + 88^\circ + m\angle J = 180^\circ$ and $m\angle J$ is also 57°.

Notice that we were given that $m\angle A = m\angle H$ and $m\angle B = m\angle I$ and we found out that $m\angle C = m\angle J$. This can be generalized into the Third Angle Theorem.

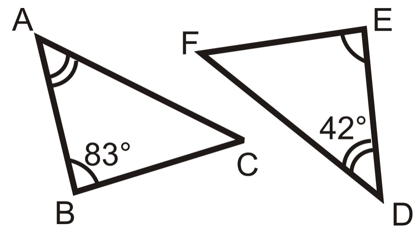
Third Angle Theorem: If two angles in one triangle are congruent to two angles in another triangle, then the third pair of angles must also congruent.

In other words, for triangles $\triangle ABC$ and $\triangle DEF, \angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\angle C \cong \angle F$.

Notice that this theorem does not state that the triangles are congruent. That is because if two sets of angles are congruent, the sides could be different lengths. See the picture to the left.



Example 5: Determine the measure of the missing angles.



Solution: From the markings, we know that $\angle A \cong D$ and $\angle E \cong \angle B$. Therefore, the Third Angle Theorem tells us that $\angle C \cong \angle F$. So,

$$\begin{aligned} m\angle A + m\angle B + m\angle C &= 180^\circ \\ m\angle D + m\angle B + m\angle C &= 180^\circ \\ 42^\circ + 83^\circ + m\angle C &= 180^\circ \\ m\angle C &= 55^\circ = m\angle F \end{aligned}$$

Congruence Properties

Recall the Properties of Congruence from Chapter 2. They will be very useful in the upcoming sections.

Reflexive Property of Congruence: Any shape is congruent to itself.

$\overline{AB} \cong \overline{AB}$ or $\triangle ABC \cong \triangle ABC$

Symmetric Property of Congruence: If two shapes are congruent, the statement can be written with either shape on either side of the $\cong$ sign.

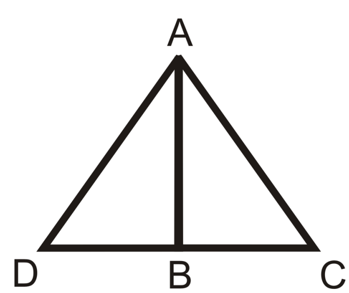
$\angle EFG \cong \angle XYZ$ and $\angle XYZ \cong \angle EFG$ or $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle ABC$

Transitive Property of Congruence: If two shapes are congruent and one of those is congruent to a third, the first and third shapes are also congruent.

$\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle GHI$, then $\triangle ABC \cong \triangle GHI$

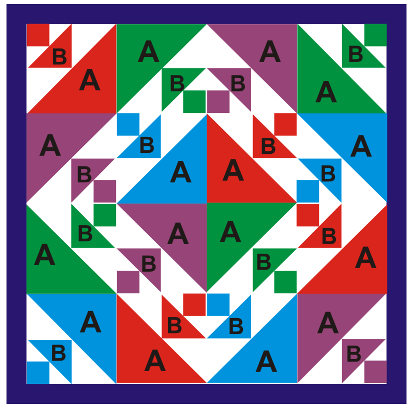
These three properties will be very important when you begin to prove that two triangles are congruent.

Example 6: In order to say that $\triangle ABD \cong \triangle ABC$, you must determine that the three corresponding angles and sides are congruent. Which pair of sides is congruent by the Reflexive Property?



Solution: The side $\overline{AB}$ is shared by both triangles. So, in a geometric proof, $\overline{AB} \cong \overline{AB}$ by the Reflexive Property of Congruence.

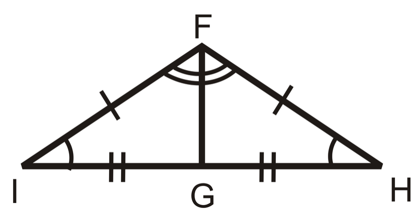
Know What? Revisited There are 16 "*A*" triangles and they are all congruent. There are 16 "*B*" triangles and they are all congruent. The quilt pattern is made from dividing up the square into smaller squares. The "*A*" triangles are all $\frac{1}{32}$ of the overall square and the "*B*" triangles are each $\frac{1}{128}$ of the large square. Both the "*A*" and "*B*" triangles are right triangles.



Review Questions

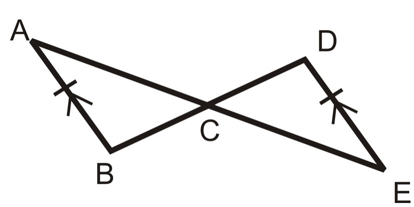
1. If $\triangle RAT \cong \triangle UGH$, what is also congruent?
2. If $\triangle BIG \cong \triangle TOP$, what is also congruent?

For questions 3-7, use the picture to the right.



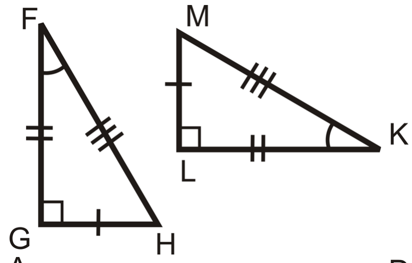
3. What theorem tells us that $\angle FGH \cong \angle FGI$?
4. What is $m\angle FGI$ and $m\angle FGH$? How do you know?
5. What property tells us that the third side of each triangle is congruent?
6. How does $\overline{FG}$ relate to $\angle IFH$?
7. Write the congruence statement for these two triangles.

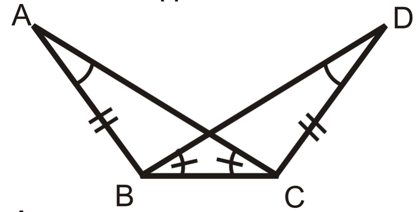
For questions 8-12, use the picture to the right.

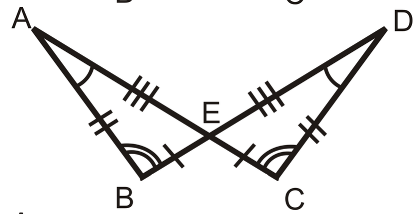


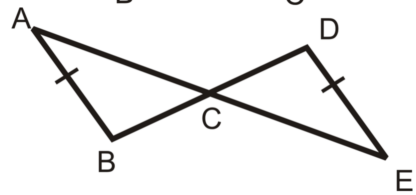
8. If $\overline{AB} \parallel \overline{DE}$, what angles are congruent? How do you know?
9. Why is $\angle ACB \cong \angle ECD$? It is not the same reason as #8.
10. Are the two triangles congruent with the information you currently have? Why or why not?
11. If you are told that C is the midpoint of $\overline{AE}$ and $\overline{BD}$, what segments are congruent?
12. Write a congruence statement for the two triangles.

For questions 13-16, determine if the triangles are congruent. If they are, write the congruence statement.

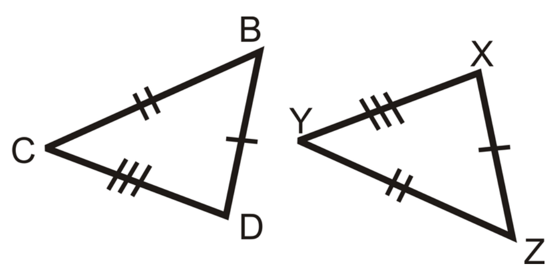
13. 

14. 

15. 

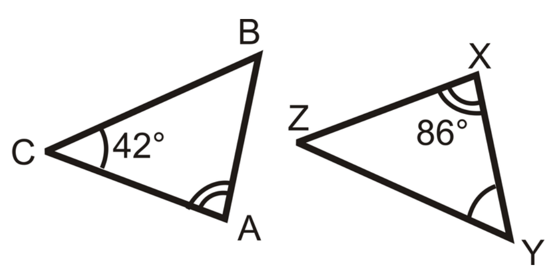
16. 

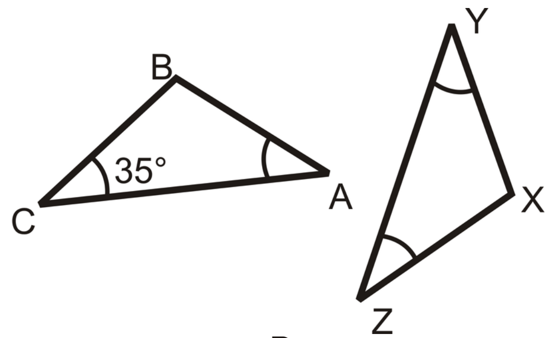
17. Suppose the two triangles to the right are congruent. Write a congruence statement for these triangles.

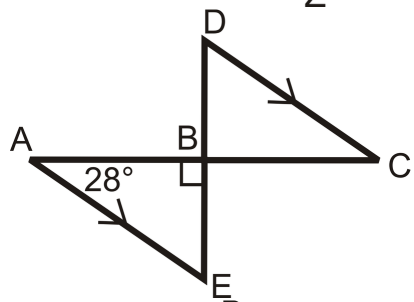


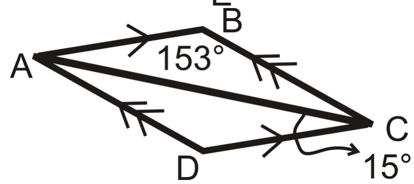
18. Explain how we know that if the two triangles are congruent, then $\angle B \cong \angle Z$.

For questions 19-22, determine the measure of all the angles in the each triangle.

19. 

20. 

21. 

22. 

23. Fill in the blanks in the Third Angle Theorem proof below. <u>Given</u>: $\angle A \cong \angle D, \angle B \cong \angle E$<u>Prove</u>: $\angle C \cong \angle F$

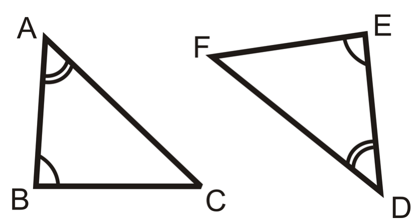


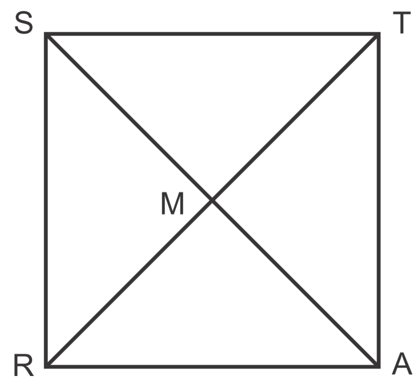
TABLE 4.4:

Statement	***Reason***
1. $A \cong \angle D, \angle B \cong \angle E$	
2.	$\cong$ angles have = measures
3. $m\angle A + m\angle B + m\angle C = 180^\circ$ m $\angle D + m\angle E + m\angle F = 180^\circ$	
4.	Substitution PoE
5.	Substitution PoE
6. $m\angle C = m\angle F$	
7. $\angle C \cong \angle F$	

For each of the following questions, determine if the Reflexive, Symmetric or Transitive Properties of Congruence is used.

24. $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$
25. $\overline{AB} \cong \overline{AB}$
26. $\triangle XYZ \cong \triangle LMN$ and $\triangle LMN \cong \triangle XYZ$
27. $\triangle ABC \cong \triangle BAC$
28. What type of triangle is $\triangle ABC$ in #27? How do you know?

Use the following diagram for questions 29 and 30.



29. Mark the diagram with the following information. $\overline{ST} \parallel \overline{RA}; \overline{SR} \parallel \overline{TA}; \overline{ST} \perp \overline{TA}$ and $\overline{SR}; \overline{SA}$ and $\overline{RT}$ are perpendicularly bisect each other.
30. Using the given information and your markings, name all of the congruent triangles in the diagram.

Review Queue Answers

1. $\angle B \cong \angle H, \overline{AB} \cong \overline{GH}, \overline{BC} \cong \overline{HI}$
2. $\angle C \cong \angle M, \overline{BC} \cong \overline{LM}$
3. The angles add up to 180°

$$\begin{aligned}(5x+2)^\circ + (4x+3)^\circ + (3x-5)^\circ &= 180^\circ \\ 12x &= 180^\circ \\ x &= 15^\circ\end{aligned}$$

4.3 Triangle Congruence using SSS and SAS

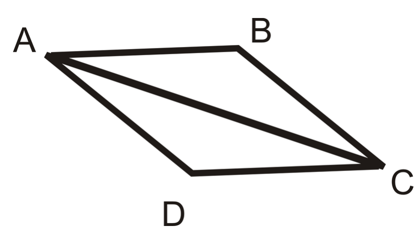
Learning Objectives

- Use the distance formula to analyze triangles on the $x-y$ plane.
- Apply the SSS Postulate to prove two triangles are congruent.
- Apply the SAS Postulate to prove two triangles are congruent.

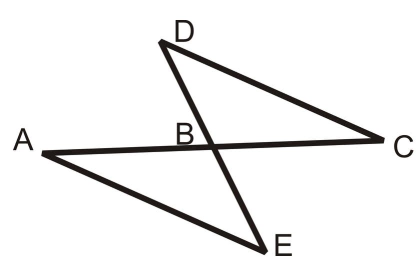
Review Queue

a. Determine the distance between the two points.
   a. (-1, 5) and (4, 12)
   b. (-6, -15) and (-3, 8)

b. Are the two triangles congruent? Explain why or why not.
   a. $\overline{AB} \mid\mid \overline{CD}, \overline{AD} \mid\mid \overline{BC}$
      $\overline{AB} \cong \overline{CD}, \overline{AD} \cong \overline{BC}$

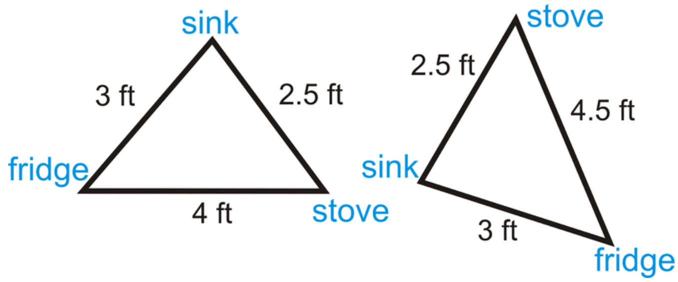


   b. B is the midpoint of $\overline{AC}$ and $\overline{DE}$



c. At this point in time, how many angles and sides do we have to know are congruent in order to say that two triangles are congruent?

Know What? The "ideal" measurements in a kitchen from the sink, refrigerator and oven are as close to an equilateral triangle as possible. Your parents are remodeling theirs to be as close to this as possible and the measurements are in the picture at the left, below. Your neighbor's kitchen has the measurements on the right. Are the two triangles congruent? Why or why not?



SSS Postulate of Triangle Congruence

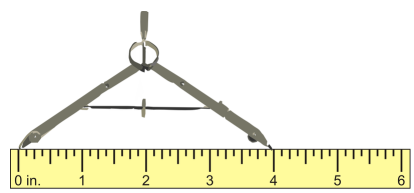
Consider the question: If I have three lengths, 3 in, 4 in, and 5 in, can I construct more than one triangle with these measurements? In other words, can I construct two different triangles with these same three lengths?

Investigation 4-2: Constructing a Triangle Given Three Sides

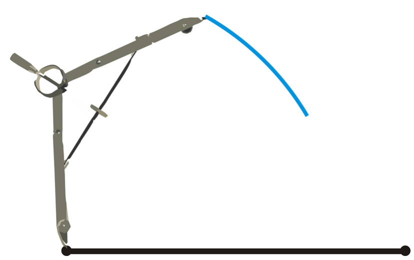
Tools Needed: compass, pencil, ruler, and paper

a. Draw the longest side (5 in) horizontally, halfway down the page. *The drawings in this investigation are to scale.*

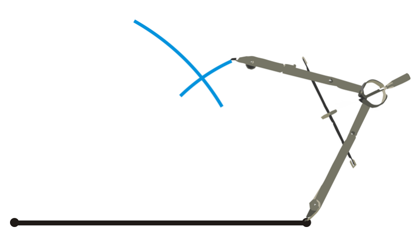
b. Take the compass and, using the ruler, widen the compass to measure 4 in, the next side.



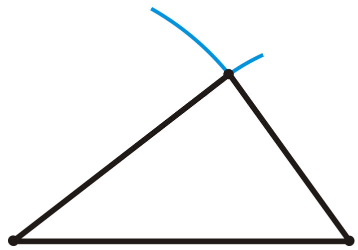
c. Using the measurement from Step 2, place the pointer of the compass on the left endpoint of the side drawn in Step 1. Draw an arc mark above the line segment.



d. Repeat Step 2 with the last measurement, 3 in. Then, place the pointer of the compass on the right endpoint of the side drawn in Step 1. Draw an arc mark above the line segment. Make sure it intersects the arc mark drawn in Step 3.



e. Draw lines from each endpoint to the arc intersections. These lines will be the other two sides of the triangle.



Can you draw another triangle, with these measurements that looks different? The answer is NO. ***Only one triangle can be created from any given three lengths.***

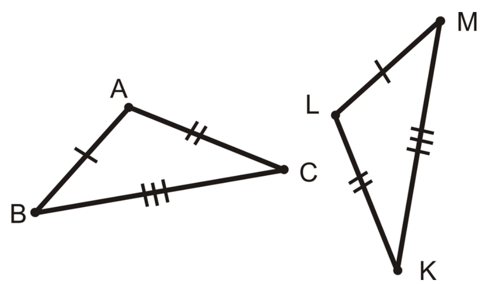
An animation of this investigation can be found at: http://www.mathsisfun.com/geometry/construct-ruler-compass-1.html

Side-Side-Side (SSS) Triangle Congruence Postulate: If three sides in one triangle are congruent to three sides in another triangle, then the triangles are congruent.

Now, we only need to show that all three sides in a triangle are congruent to the three sides in another triangle. This is a postulate so we accept it as true without proof.

Think of the SSS Postulate as a shortcut. You no longer have to show 3 sets of angles are congruent and 3 sets of sides are congruent in order to say that the two triangles are congruent.

Example 1: Write a triangle congruence statement based on the diagram below:

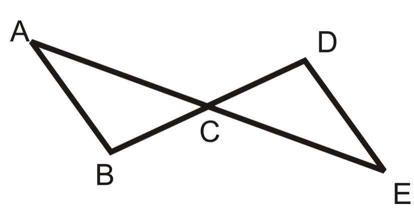


Solution: From the tic marks, we know $\overline{AB} \cong \overline{LM}, \overline{AC} \cong \overline{LK}, \overline{BC} \cong \overline{MK}$. Using the SSS Postulate we know the two triangles are congruent. Lining up the corresponding sides, we have $\triangle ABC \cong \triangle LMK$.

Don't forget ORDER MATTERS when writing triangle congruence statements. Here, we lined up the sides with one tic mark, then the sides with two tic marks, and finally the sides with three tic marks.

Example 2: Write a two-column proof to show that the two triangles are congruent.

Given: $\overline{AB} \cong \overline{DE}$



C is the midpoint of $\overline{AE}$ and $\overline{DB}$.

Prove: $\triangle ACB \cong \triangle ECD$

Solution:

TABLE 4.5:

Statement	*Reason*
1. $\overline{AB} \cong \overline{DE}$	Given
C is the midpoint of $\overline{AE}$ and $\overline{DB}$	
2. $\overline{AC} \cong \overline{CE}, \overline{BC} \cong \overline{CD}$	Definition of a midpoint
3. $\triangle ACB \cong \triangle ECD$	SSS Postulate

Make sure that you clearly state the three sets of congruent sides BEFORE stating that the triangles are congruent.

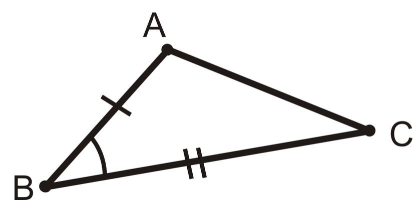
Prove Move: Feel free to mark the picture with the information you are given as well as information that you can infer (vertical angles, information from parallel lines, midpoints, angle bisectors, right angles).

SAS Triangle Congruence Postulate

First, it is important to note that SAS refers to Side-Angle-Side. The placement of the word Angle is important because it indicates that the angle that you are given is between the two sides.

Included Angle: When an angle is between two given sides of a triangle (or polygon).

In the picture to the left, the markings indicate that $\overline{AB}$ and $\overline{BC}$ are the given sides, so $\angle B$ would be the included angle.

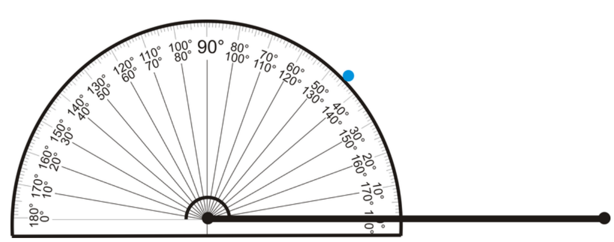


Consider the question: If I have two sides of length 2 in and 5 in and the angle between them is 45°, can I construct only one triangle?

Investigation 4-3: Constructing a Triangle Given Two Sides and Included Angle Tools Needed: protractor, pencil, ruler, and paper

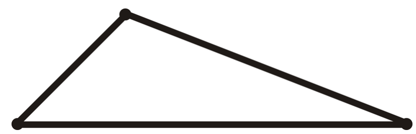
a. Draw the longest side (5 in) horizontally, halfway down the page. *The drawings in this investigation are to scale.*

b. At the left endpoint of your line segment, use the protractor to measure a 45° angle. Mark this measurement.



c. Connect your mark from Step 2 with the left endpoint. Make your line 2 in long, the length of the second side.

d. Connect the two endpoints by drawing the third side.

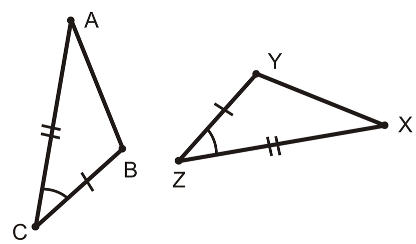


Can you draw another triangle, with these measurements that looks different? The answer is NO. ***Only one triangle can be created from any given two lengths and the INCLUDED angle.***

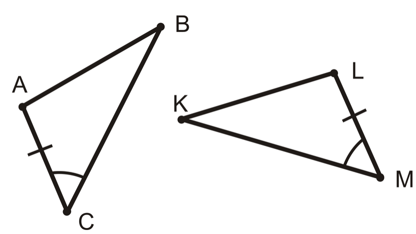
Side-Angle-Side (SAS) Triangle Congruence Postulate: If two sides and the included angle in one triangle are congruent to two sides and the included angle in another triangle, then the two triangles are congruent.

The markings in the picture are enough to say that $\triangle ABC \cong \triangle XYZ$.

So, in addition to SSS congruence, we now have SAS. Both of these postulates can be used to say that two triangles are congruent. When doing proofs, you might be able to use either SSS or SAS to prove that two triangles are congruent. There is no set way to complete a proof, so when faced with the choice to use SSS or SAS, it does not matter. Either would be correct.



Example 3: What additional piece of information would you need to prove that these two triangles are congruent using the SAS Postulate?



a) $\angle ABC \cong \angle LKM$

b) $\overline{AB} \cong \overline{LK}$

c) $\overline{BC} \cong \overline{KM}$

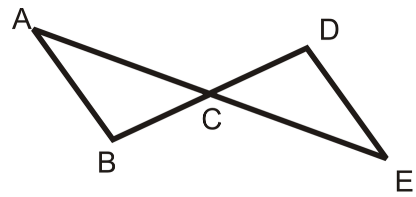
d) $\angle BAC \cong \angle KLM$

Solution: For the SAS Postulate, you need two sides and the included angle in both triangles. So, you need the side on the other side of the angle. In $\triangle ABC$, that is $\overline{BC}$ and in $\triangle LKM$ that is $\overline{KM}$. The correct answer is c.

Example 4: Write a two-column proof to show that the two triangles are congruent.

Given: C is the midpoint of $\overline{AE}$ and $\overline{DB}$

Prove: $\triangle ACB \cong \triangle ECD$



Solution:

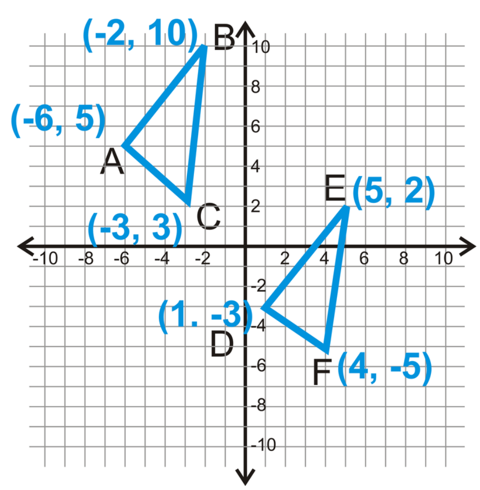
TABLE 4.6:

Statement	***Reason***
1. C is the midpoint of $\overline{AE}$ and $\overline{DB}$	Given
2. $\overline{AC} \cong \overline{CE}, \overline{BC} \cong \overline{CD}$	Definition of a midpoint
3. $\angle ACB \cong \angle DCE$	Vertical Angles Postulate
4. $\triangle ACB \cong \triangle ECD$	SAS Postulate

In Example 4, we could have only proven the two triangles congruent by SAS. If we were given that $\overline{AB} \cong \overline{DE}$, then we could have also proven the two triangles congruent by SSS.

SSS in the Coordinate Plane

In the coordinate plane, the easiest way to show two triangles are congruent is to find the lengths of the 3 sides in each triangle. Finding the measure of an angle in the coordinate plane can be a little tricky, so we will avoid it in this text. Therefore, you will only need to apply SSS in the coordinate plane. To find the lengths of the sides, you will need to use the distance formula, $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.



Example 5: Find the distances of all the line segments from both triangles to see if the two triangles are congruent.

Solution: Begin with $\triangle ABC$ and its sides.

$$\begin{aligned}AB &= \sqrt{(-6-(-2))^2+(5-10)^2}\\ &= \sqrt{(-4)^2+(-5)^2}\\ &= \sqrt{16+25}\\ &= \sqrt{41}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(-2-(-3))^2+(10-3)^2}\\ &= \sqrt{(1)^2+(7)^2}\\ &= \sqrt{1+49}\\ &= \sqrt{50}=5\sqrt{2}\end{aligned}$$

$$\begin{aligned}AC &= \sqrt{(-6-(-3))^2+(5-3)^2}\\ &= \sqrt{(-3)^2+(2)^2}\\ &= \sqrt{9+4}\\ &= \sqrt{13}\end{aligned}$$

Now, find the distances of all the sides in $\triangle DEF$.

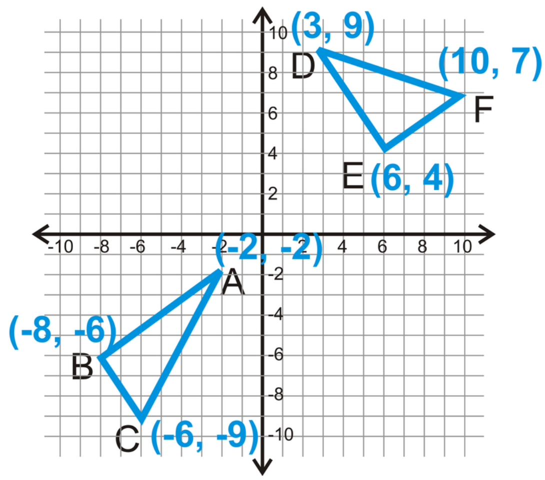
$$\begin{aligned}DE &= \sqrt{(1-5)^2+(-3-2)^2}\\ &= \sqrt{(-4)^2+(-5)^2}\\ &= \sqrt{16+25}\\ &= \sqrt{41}\end{aligned}$$

$$\begin{aligned}EF &= \sqrt{(5-4)^2+(2-(-5))^2}\\ &= \sqrt{(1)^2+(7)^2}\\ &= \sqrt{1+49}\\ &= \sqrt{50}=5\sqrt{2}\end{aligned}$$

$$\begin{aligned}DF &= \sqrt{(1-4)^2+(-3-(-5))^2}\\ &= \sqrt{(-3)^2+(2)^2}\\ &= \sqrt{9+4}\\ &= \sqrt{13}\end{aligned}$$

We see that $AB = DE, BC = EF$, and $AC = DF$. Recall that if two lengths are equal, then they are also congruent. Therefore, $\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$. Because the corresponding sides are congruent, we can say that $\triangle ABC \cong \triangle DEF$ by SSS.

Example 6: Determine if the two triangles are congruent.



Solution: Use the distance formula to find all the lengths. Start with $\triangle ABC$.

$$\begin{aligned} AB &= \sqrt{(-2-(-8))^2+(-2-(-6))^2} \\ &= \sqrt{(6)^2+(4)^2} \\ &= \sqrt{36+16} \\ &= \sqrt{52} = 2\sqrt{13} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(-8-(-6))^2+(-6-(-9))^2} \\ &= \sqrt{(-2)^2+(3)^2} \\ &= \sqrt{4+9} \\ &= \sqrt{13} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(-2-(-6))^2+(-2-(-9))^2} \\ &= \sqrt{(-4)^2+(7)^2} \\ &= \sqrt{16+49} \\ &= \sqrt{65} \end{aligned}$$

Now find the sides of $\triangle DEF$.

$$\begin{aligned}DE &= \sqrt{(3-6)^2+(9-4)^2}\\ &= \sqrt{(-3)^2+(5)^2}\\ &= \sqrt{9+25}\\ &= \sqrt{34}\end{aligned}$$

$$\begin{aligned}EF &= \sqrt{(6-10)^2+(4-7)^2}\\ &= \sqrt{(-4)^2+(-3)^2}\\ &= \sqrt{16+9}\\ &= \sqrt{25}=5\end{aligned}$$

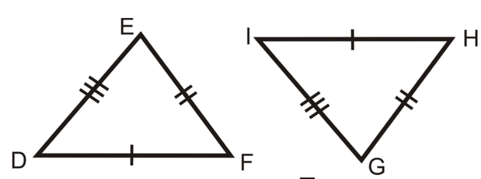
$$\begin{aligned}DF &= \sqrt{(3-10)^2+(9-7)^2}\\ &= \sqrt{(-7)^2+(2)^2}\\ &= \sqrt{49+4}\\ &= \sqrt{53}\end{aligned}$$

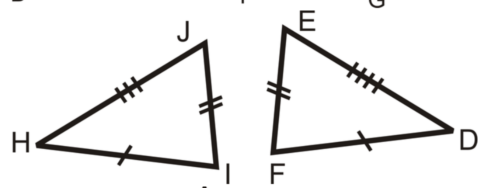
No sides have equal measures, so the triangles are not congruent.

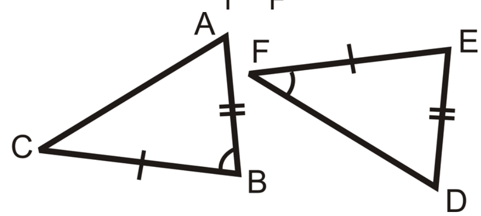
Know What? Revisited From what we have learned in this section, the two triangles are not congruent because the distance from the fridge to the stove in your house is 4 feet and in your neighbor's it is 4.5 ft. The SSS Postulate tells us that all three sides have to be congruent.

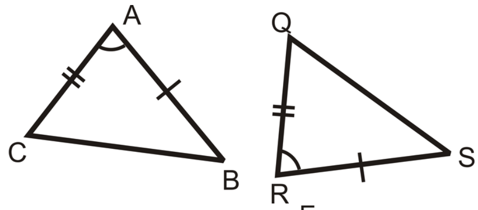
Review Questions

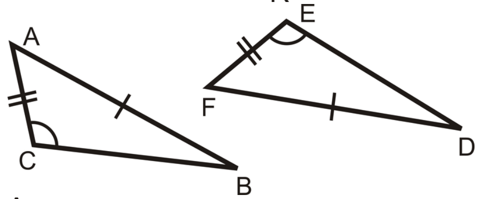
Are the pairs of triangles congruent? If so, write the congruence statement.

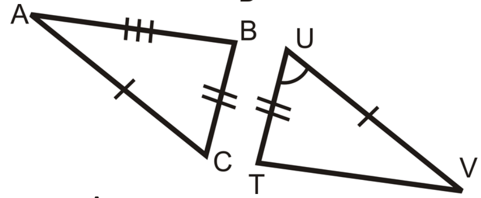
1. 

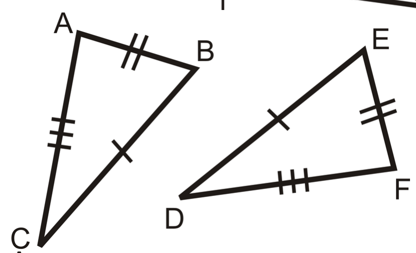
2. 

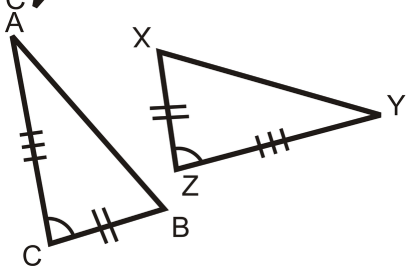
3. 

4. 

5. 

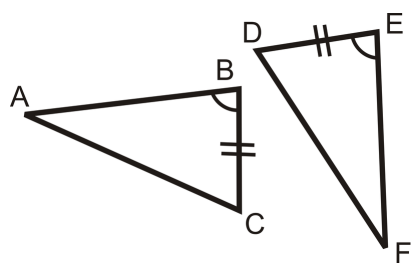
6. 

7. 

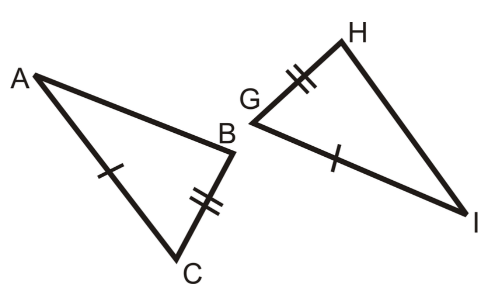
8. 

State the additional piece of information needed to show that each pair of triangles are congruent.

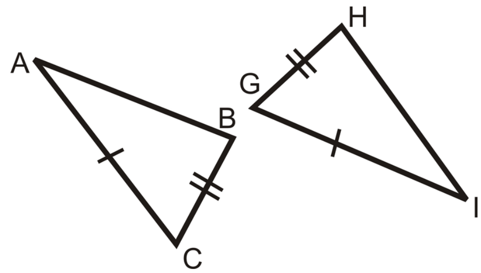
9. Use SAS



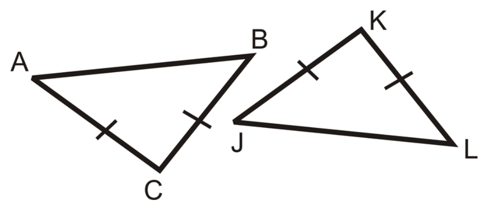
10. Use SSS



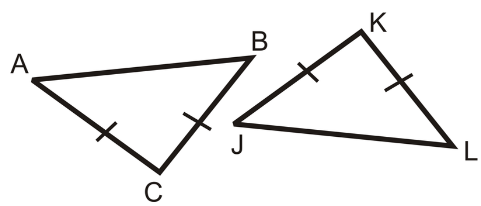
11. Use SAS



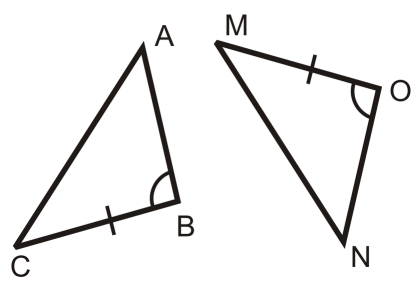
12. Use SAS



13. Use SSS



14. Use SAS



Fill in the blanks in the proofs below.

15. Given: $\overline{AB} \cong \overline{DC}, \overline{BE} \cong \overline{CE}$ Prove: $\triangle ABE \cong \triangle ACE$

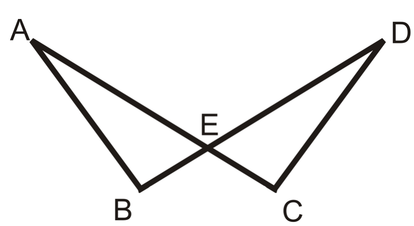


TABLE 4.7:

Statement	***Reason***
1.	1.
2. $\angle AEB \cong \angle DEC$	2.
3. $\triangle ABE \cong \triangle ACE$	3.

16. Given: $\overline{AB} \cong \overline{DC}, \overline{AC} \cong \overline{DB}$Prove: $\triangle ABC \cong \triangle DCB$

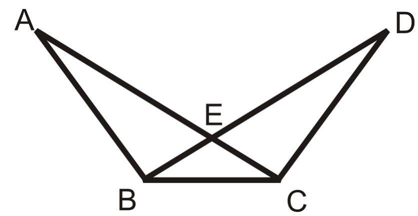


TABLE 4.8:

Statement	Reason
1.	1.
2.	2. Reflexive PoC
3. $\triangle ABC \cong \triangle DCB$	3.

17. Given: B is a midpoint of $\overline{DC}\overline{AB} \perp \overline{DC}$Prove: $\triangle ABD \cong \triangle ABC$

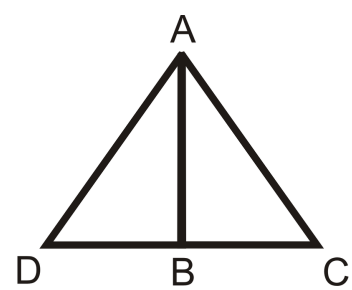
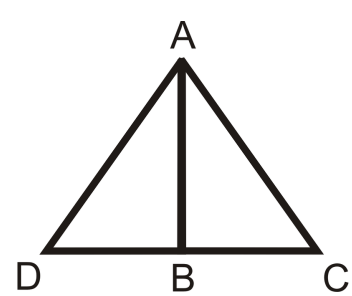


TABLE 4.9:

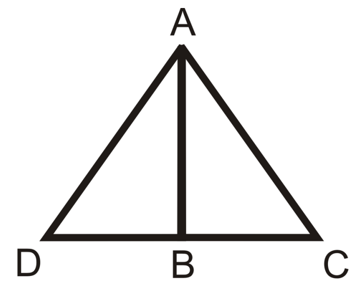
Statement	Reason
1. B is a midpoint of $\overline{DC}, \overline{AB} \perp \overline{DC}$	1.
2.	2. Definition of a midpoint
3. $\angle ABD$ and $\angle ABC$ are right angles	3.
4.	4. All right angles are $\cong$
5.	5.
6. $\triangle ABD \cong \triangle ABC$	6.

Write a two-column proof for the given information below.

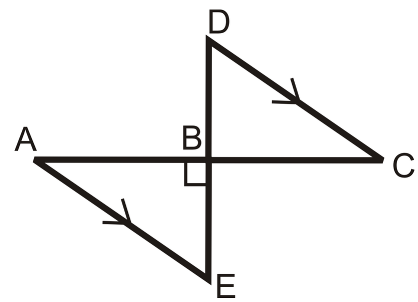
18. Given: $\overline{AB}$ is an angle bisector of $\angle DAC\overline{AD} \cong \overline{AC}$Prove: $\triangle ABD \cong \triangle ABC$



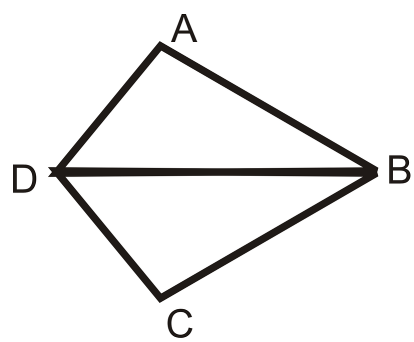
19. Given: B is the midpoint of $\overline{DC}\overline{AD} \cong \overline{AC}$Prove: $\triangle ABD \cong \triangle ABC$



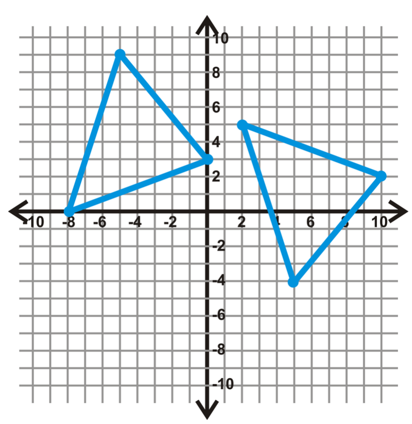
20. Given: B is the midpoint of $\overline{DE}$ and $\overline{AC} \angle ABE$ is a right angle Prove: $\triangle ABE \cong \triangle CBD$



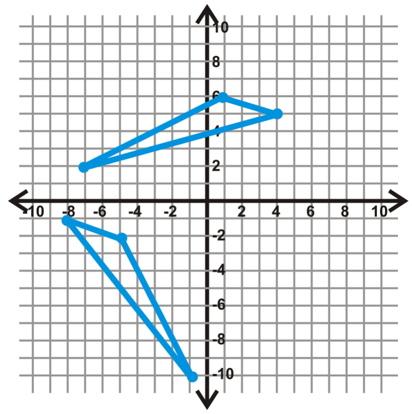
21. Given: $\overline{DB}$ is the angle bisector of $\angle ADC \overline{AD} \cong \overline{DC}$ Prove: $\triangle ABD \cong \triangle CBD$



Determine if the two triangles are congruent, using the distance formula. Leave all of your answers in simplest radical form (simplify all radicals, no decimals).



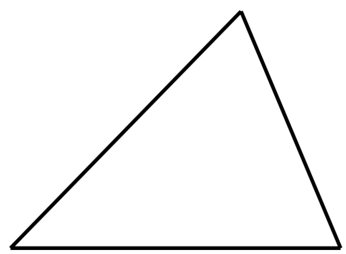
22.

23. 

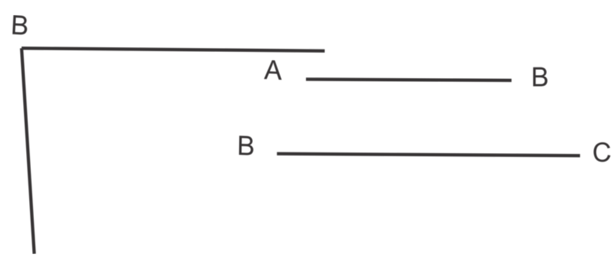
24. $\triangle ABC : A(-1,5), B(-4,2), C(2,-2)$ and $\triangle DEF : D(7,-5), E(4,2), F(8,-9)$
25. $\triangle ABC : A(-8,-3), B(-2,-4), C(-5,-9)$ and $\triangle DEF : D(-7,2), E(-1,3), F(-4,8)$

Constructions

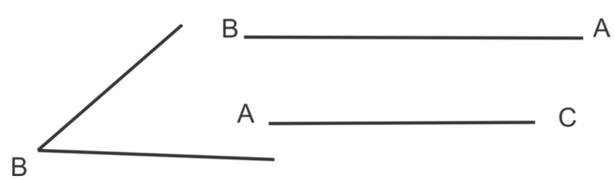
26. Construct a triangle with sides of length 5cm, 3cm, 2cm.
27. Copy the triangle below using a straightedge and compass.



28. Use the two sides and the given angle to construct $\triangle ABC$.



29. Use the two sides and the given angle to construct $\triangle ABC$.



30. Was the information given in problem 29 in SAS order? If not, your triangle may not be the only triangle that you can construct using the given information. Construct the second possible triangle.

Review Queue Answers

a. a. $\sqrt{74}$

b. $\sqrt{538}$

a. Yes, $\triangle CAD \cong \triangle ACB$ because $\angle CAD \cong \angle ACB$ and $\angle BAC \cong \angle ACD$ by Alternate Interior Angles. $\overline{AC} \cong \overline{AC}$ by the Reflexive PoC and $\angle ADC \cong \angle ABC$ by the 3^{rd} Angle Theorem.

b. At this point in time, we do not have enough information to show that the two triangles are congruent. We know that $\overline{AB} \cong \overline{BC}$ and $\overline{DB} \cong \overline{BE}$ from the definition of a midpoint. By vertical angles, we know that $\angle DBC \cong \angle ABE$. This is only two sides and one pair of angles; not enough info, yet.

b. We need to know three pairs of congruent sides and two pairs of congruent angles. From this, we can assume the third pair of angles are congruent from the 3^{rd} Angle Theorem.

4.4 Triangle Congruence Using ASA, AAS, and HL

Learning Objectives

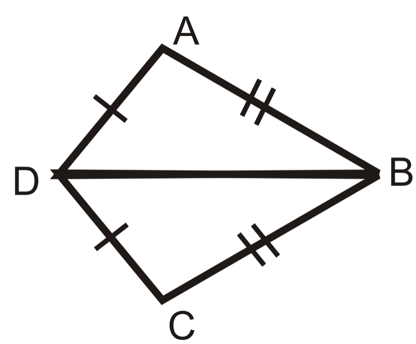
- Use the ASA Congruence Postulate, AAS Congruence Theorem, and the HL Congruence Theorem.
- Complete two-column proofs using SSS, SAS, ASA, AAS, and HL.

Review Queue

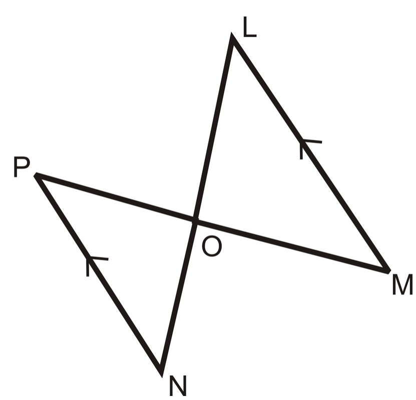
1. Write a two-column proof.

Given: $\overline{AD} \cong \overline{DC}, \overline{AB} \cong \overline{CB}$

Prove: $\triangle DAB \cong \triangle DCB$



2. Is $\triangle PON \cong \triangle MOL$? Why or why not?

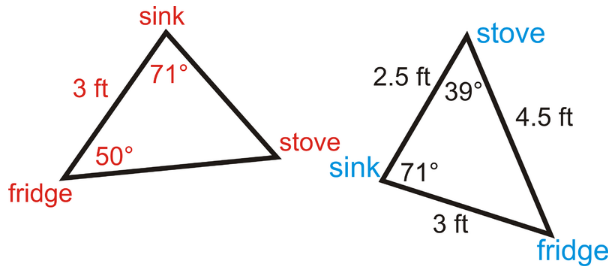


3. If $\triangle DEF \cong \triangle PQR$, can it be assumed that:

a) $\angle F \cong \angle R$? Why or why not?

b) $\overline{EF} \cong \overline{PR}$? Why or why not?

Know What? Your parents changed their minds at the last second about their kitchen layout. Now, they have decided they to have the distance between the sink and the fridge be 3 ft, the angle at the sink 71° and the angle at the fridge is 50°. You used your protractor to measure the angle at the stove and sink at your neighbor's house. Are the kitchen triangles congruent now?



ASA Congruence

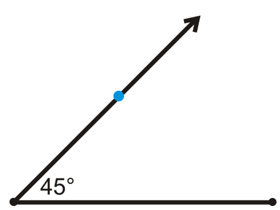
Like SAS, ASA refers to Angle-Side-Angle. The placement of the word Side is important because it indicates that the side that you are given is between the two angles.

Consider the question: If I have two angles that are 45° and 60° and the side between them is 5 in, can I construct only one triangle? We will investigate it here.

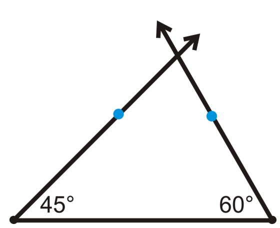
Investigation 4-4: Constructing a Triangle Given Two Angles and Included Side Tools Needed: protractor, pencil, ruler, and paper

a. Draw the side (5 in) horizontally, halfway down the page. *The drawings in this investigation are to scale.*

b. At the left endpoint of your line segment, use the protractor to measure the 45° angle. Mark this measurement and draw a ray from the left endpoint through the 45° mark.



c. At the right endpoint of your line segment, use the protractor to measure the 60° angle. Mark this measurement and draw a ray from the left endpoint through the 60° mark. Extend this ray so that it crosses through the ray from Step 2.

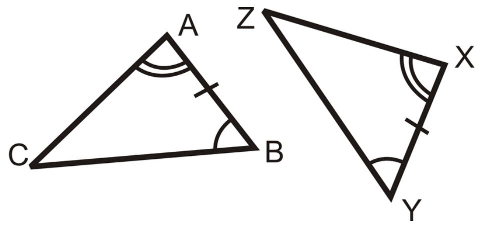


d. Erase the extra parts of the rays from Steps 2 and 3 to leave only the triangle.

Can you draw another triangle, with these measurements that looks different? The answer is NO. ***Only one triangle can be created from any given two angle measures and the INCLUDED side.***

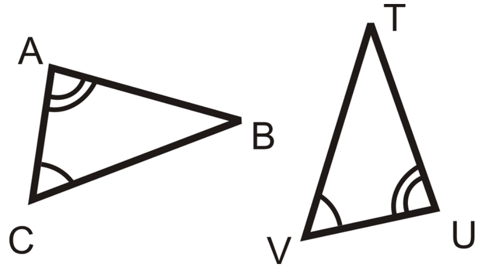
Angle-Side-Angle (ASA) Congruence Postulate: If two angles and the included side in one triangle are congruent to two angles and the included side in another triangle, then the two triangles are congruent.

The markings in the picture are enough to say $\triangle ABC \cong \triangle XYZ$.



Now, in addition to SSS and SAS, you can use ASA to prove that two triangles are congruent.

Example 1: What information would you need to prove that these two triangles are congruent using the ASA Postulate?



a) $\overline{AB} \cong \overline{UT}$

b) $\overline{AC} \cong \overline{UV}$

c) $\overline{BC} \cong \overline{TV}$

d) $\angle B \cong \angle T$

Solution: For ASA, we need the side between the two given angles, which is $\overline{AC}$ and $\overline{UV}$. The answer is b.

Example 2: Write a 2-column proof.

Given: $\angle C \cong \angle E, \overline{AC} \cong \overline{AE}$

Prove: $\triangle ACF \cong \triangle AEB$

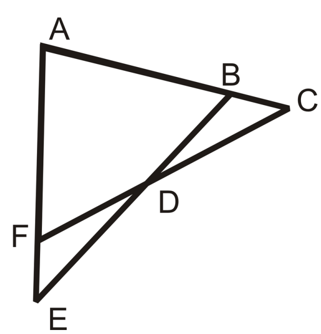
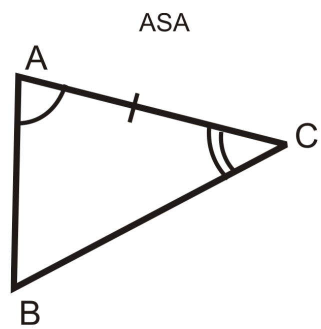


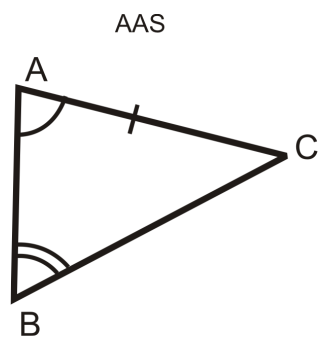
TABLE 4.10:

Statement	*Reason*
1. $\angle C \cong \angle E, \overline{AC} \cong \overline{AE}$	Given
2. $\angle A \cong \angle A$	Reflexive PoC
3. $\triangle ACF \cong \triangle AEB$	ASA

AAS Congruence

A variation on ASA is AAS, which is Angle-Angle-Side. Recall that for ASA you need two angles and the side between them. But, if you know two pairs of angles are congruent, then the third pair will also be congruent by the 3^{rd} Angle Theorem. Therefore, you can prove a triangle is congruent whenever you have any two angles and a side.





Be careful to note the placement of the side for ASA and AAS. As shown in the pictures above, the side is ***between*** the two angles for ASA and it is not for AAS.

Angle-Angle-Side (AAS or SAA) Congruence Theorem: If two angles and a non-included side in one triangle are congruent to two corresponding angles and a non-included side in another triangle, then the triangles are congruent.

Proof of AAS Theorem:

Given: $\angle A \cong \angle Y, \angle B \cong \angle Z, \overline{AC} \cong \overline{XY}$

Prove: $\triangle ABC \cong \triangle YZX$

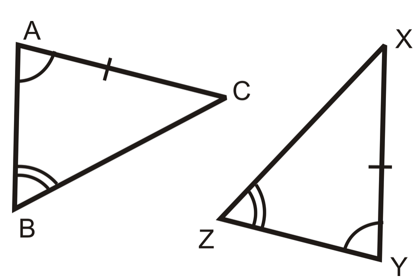


TABLE 4.11:

Statement	Reason
1. $\angle A \cong \angle Y, \angle B \cong \angle Z, \overline{AC} \cong \overline{XY}$	Given
2. $\angle C \cong \angle X$	3^{rd} Angle Theorem
3. $\triangle ABC \cong \triangle YZX$	ASA

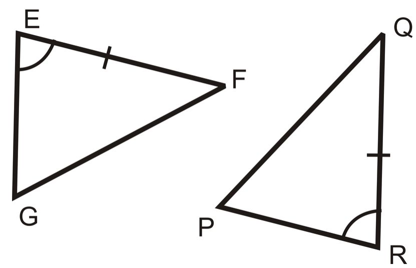
By proving $\triangle ABC \cong \triangle YZX$ with ASA, we have also shown that the AAS Theorem is valid. You can now use this theorem to show that two triangles are congruent.

Example 3: What information do you need to prove that these two triangles are congruent using:

a) ASA?

b) AAS?

c) SAS?



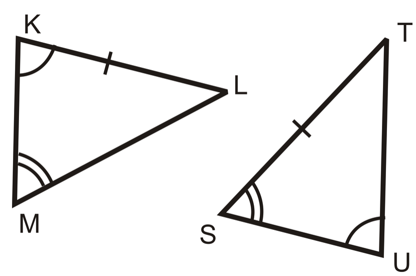
Solution:

a) For ASA, we need the angles on the other side of $\overline{EF}$ and $\overline{QR}$. Therefore, we would need $\angle F \cong \angle Q$.

b) For AAS, we would need the angle on the other side of $\angle E$ and $\angle R$. $\angle G \cong \angle P$.

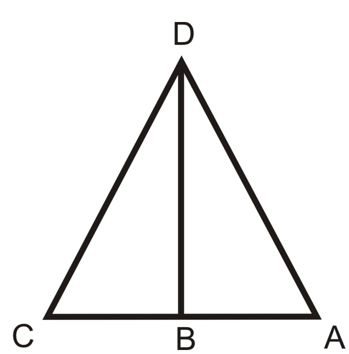
c) For SAS, we would need the side on the other ***side*** of $\angle E$ and $\angle R$. So, we would need $\overline{EG} \cong \overline{RP}$.

Example 4: Can you prove that the following triangles are congruent? Why or why not?



Solution: Even though $\overline{KL} \cong \overline{ST}$, they are not corresponding. Look at the angles around $\overline{KL}$, $\angle K$ and $\angle L$. $\angle K$ has **one** arc and $\angle L$ is unmarked. The angles around $\overline{ST}$ are $\angle S$ and $\angle T$. $\angle S$ has **two** arcs and $\angle T$ is unmarked. In order to use AAS, $\angle S$ needs to be congruent to $\angle K$. They are not congruent because the arcs marks are different. Therefore, we cannot conclude that these two triangles are congruent.

Example 5: Write a 2-column proof.



Given: $\overline{BD}$ is an angle bisector of $\angle CDA$, $\angle C \cong \angle A$

Prove: $\triangle CBD \cong \angle ABD$

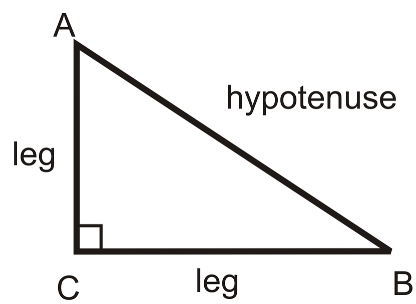
Solution:

TABLE 4.12:

Statement	*Reason*
1. $\overline{BD}$ is an angle bisector of $\angle CDA$, $\angle C \cong \angle A$	Given
2. $\angle CDB \cong \angle ADB$	Definition of an Angle Bisector
3. $\overline{DB} \cong \overline{DB}$	Reflexive PoC
3. $\triangle CBD \cong \triangle ABD$	AAS

Hypotenuse-Leg Congruence Theorem

So far, the congruence postulates we have learned will work on any triangle. The last congruence theorem can only be used on right triangles. Recall that a right triangle has exactly one right angle. The two sides adjacent to the right angle are called legs and the side opposite the right angle is called the hypotenuse.



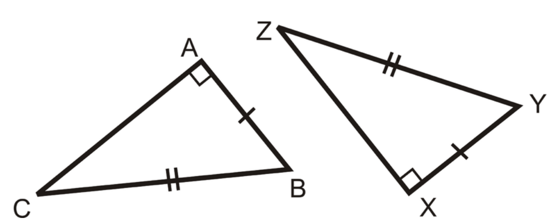
You may or may not know the Pythagorean Theorem (which will be covered in more depth later in this text). It says, for any ***right*** triangle, this equation is true:

$(leg)^2 + (leg)^2 = (hypotenuse)^2$. What this means is that if you are given two sides of a right triangle, you can always find the third.

Therefore, if you know that two sides of a ***right*** triangle are congruent to two sides of another ***right*** triangle, you can conclude that third sides are also congruent.

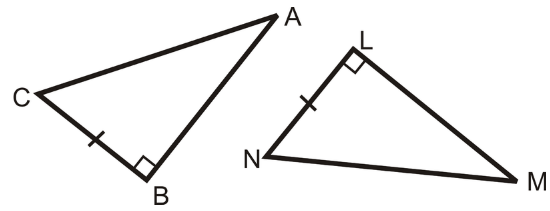
HL Congruence Theorem: If the hypotenuse and leg in one right triangle are congruent to the hypotenuse and leg in another right triangle, then the two triangles are congruent.

The markings in the picture are enough to say $\triangle ABC \cong \triangle XYZ$.



Notice that this theorem is only used with a hypotenuse and a leg. If you know that the two legs of a right triangle are congruent to two legs of another triangle, the two triangles would be congruent by SAS, because the right angle would be between them. We will not prove this theorem here because we have not proven the Pythagorean Theorem yet.

Example 6: What information would you need to prove that these two triangles are congruent using the: a) HL Theorem? b) SAS Theorem?



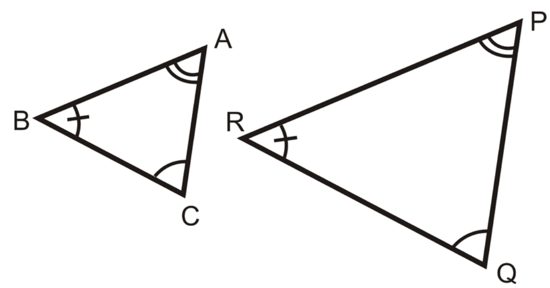
Solution:

a) For HL, you need the hypotenuses to be congruent. So, $\overline{AC} \cong \overline{MN}$.

b) To use SAS, we would need the other legs to be congruent. So, $\overline{AB} \cong \overline{ML}$.

AAA and SSA Relationships

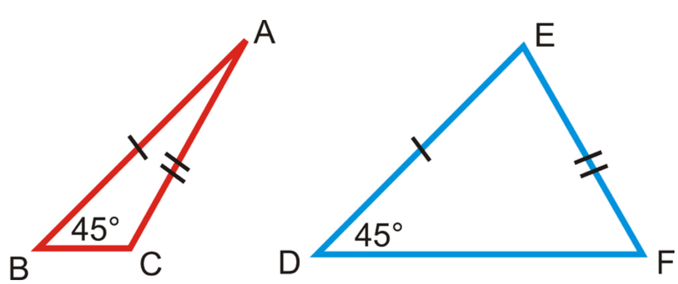
There are two other side-angle relationships that we have not discussed: AAA and SSA.



AAA implied that all the angles are congruent, however, that does not mean the triangles are congruent.

As you can see, $\triangle ABC$ and $\triangle PRQ$ are not congruent, even though all the angles are. These triangles are similar, a topic that will be discussed later in this text.

SSA relationships do not prove congruence either. In review problems 29 and 30 of the last section you illustrated an example of how SSA could produce two different triangles. $\triangle ABC$ and $\triangle DEF$ below are another example of SSA.



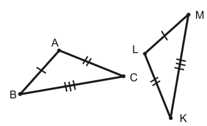
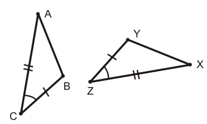
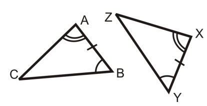
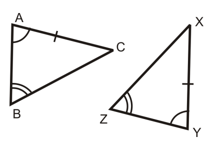
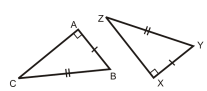
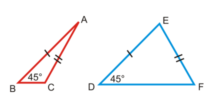
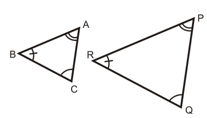
$\angle B$ and $\angle D$ are ***not*** the included angles between the congruent sides, so we cannot prove that these two triangles are congruent. Notice, that two different triangles can be drawn even though $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{EF}$, and $m\angle B = m\angle D$.

You might have also noticed that SSA could also be written ASS. This is true, however, in this text we will write SSA.

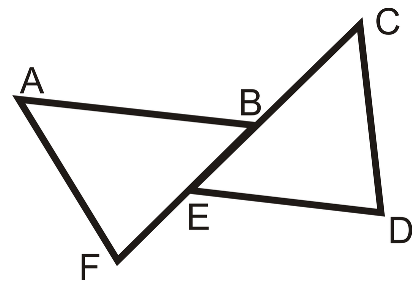
Triangle Congruence Recap

To recap, here is a table of all of the possible side-angle relationships and if you can use them to determine congruence or not.

TABLE 4.13:

Side-Angle Relationship	*Picture*	*Determine Congruence?*
SSS		**Yes** $\triangle ABC \cong \triangle LKM$
SAS		**Yes** $\triangle ABC \cong \triangle XYZ$
ASA		**Yes** $\triangle ABC \cong \triangle XYZ$
AAS (or SAA)		**Yes** $\triangle ABC \cong \triangle YZX$
HL		**Yes, Right Triangles Only** $\triangle ABC \cong \triangle XYZ$
SSA (or ASS)		**NO**
AAA		**NO**

Example 7: Write a 2-column proof.



Given: $\overline{AB} \parallel \overline{ED}, \angle C \cong \angle F, \overline{AB} \cong \overline{ED}$

Prove: $\overline{AF} \cong \overline{CD}$

Solution:

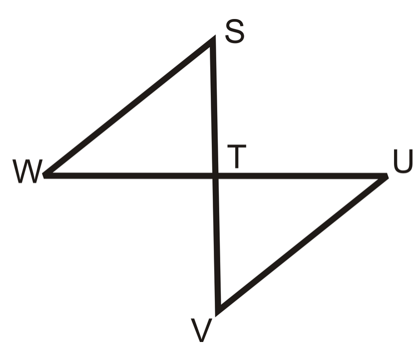
TABLE 4.14:

Statement	*Reason*
1. $\overline{AB} \parallel \overline{ED}, \angle C \cong \angle F, \overline{AB} \cong \overline{ED}$	Given
2. $\angle ABE \cong \angle DEB$	Alternate Interior Angles Theorem
3. $\triangle ABF \cong \triangle DEC$	ASA
4. $\overline{AF} \cong \overline{CD}$	CPCTC

Example 8: Write a 2-column proof.

Given: T is the midpoint of $\overline{WU}$ and $\overline{SV}$

Prove: $\overline{WS} \parallel \overline{VU}$



Solution:

TABLE 4.15:

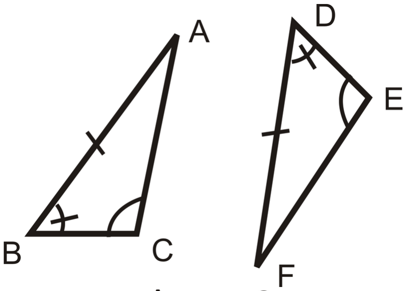
Statement	*Reason*
1. T is the midpoint of $\overline{WU}$ and $\overline{SV}$	Given
2. $\overline{WT} \cong \overline{TU}, \overline{ST} \cong \overline{TV}$	Definition of a midpoint
3. $\angle STW \cong \angle UTV$	Vertical Angle Theorem
4. $\triangle STW \cong \triangle VTU$	SAS
5. $\angle S \cong \angle V$	CPCTC
6. $\overline{WS} \parallel \overline{VU}$	Converse of the Alternate Interior Angles Theorem

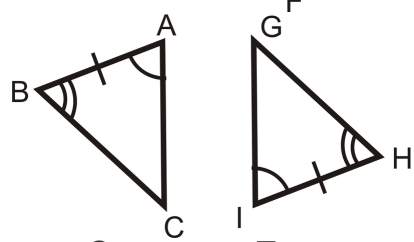
Prove Move: At the beginning of this chapter we introduced CPCTC. Now, it can be used in a proof once two triangles are proved congruent. It is used to prove the parts of congruent triangles are congruent in order to prove that sides are parallel (like in Example 8), midpoints, or angle bisectors. You will do proofs like these in the review questions.

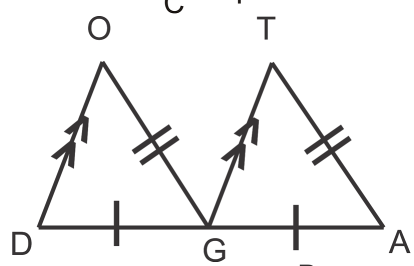
Know What? Revisited Even though we do not know all of the angle measures in the two triangles, we can find the missing angles by using the Third Angle Theorem. In your parents' kitchen, the missing angle is 39°. The missing angle in your neighbor's kitchen is 50°. From this, we can conclude that the two kitchens are now congruent, either by ASA or AAS.

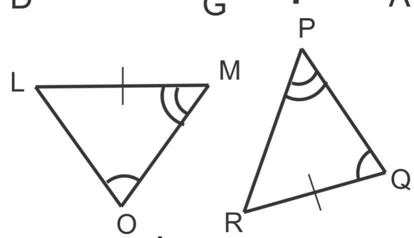
Review Questions

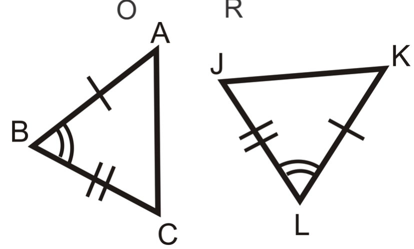
For questions 1-10, determine if the triangles are congruent. If they are, write the congruence statement and which congruence postulate or theorem you used.

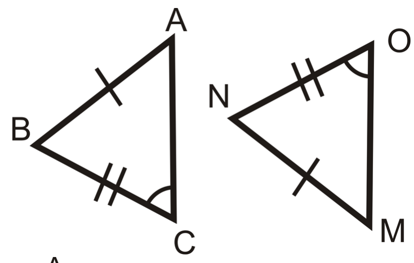
1. 

2. 

3. 

4. 

5. 



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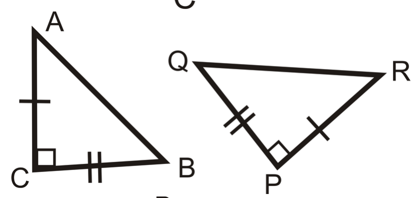

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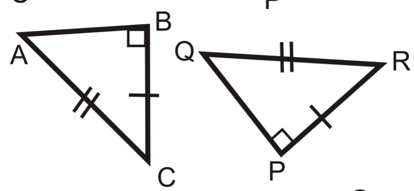

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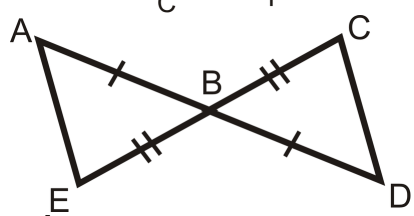

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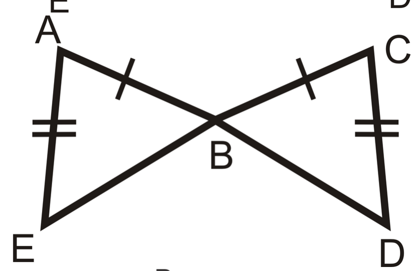

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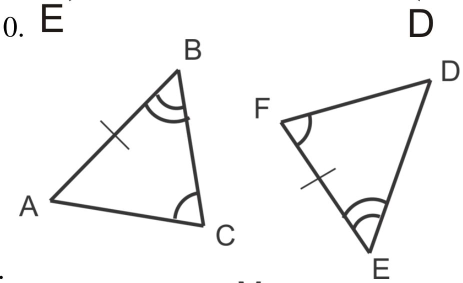

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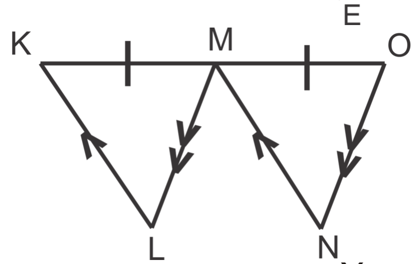

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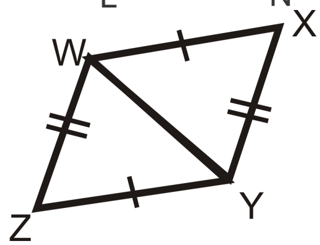

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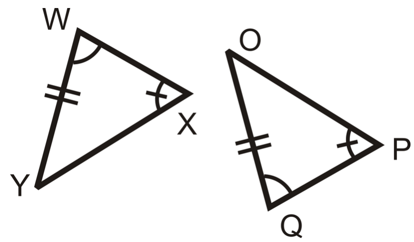
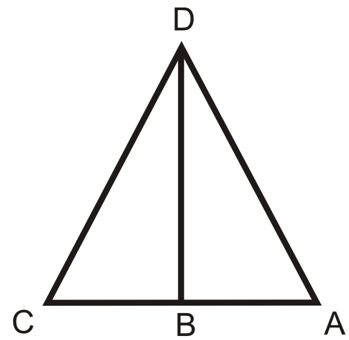


14.

For questions 11-15, use the picture to the right and the given information below.

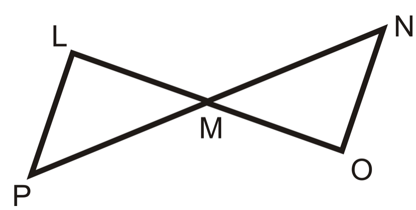


Given: $\overline{DB} \perp \overline{AC}, \overline{DB}$ is the angle bisector of $\angle CDA$

15. From $\overline{DB} \perp \overline{AC}$, which angles are congruent and why?
16. Because $\overline{DB}$ is the angle bisector of $\angle CDA$, what two angles are congruent?
17. From looking at the picture, what additional piece of information are you given? Is this enough to prove the two triangles are congruent?
18. Write a 2-column proof to prove $\triangle CDB \cong \triangle ADB$.
19. What would be your reason for $\angle C \cong \angle A$?

For questions 16-20, use the picture to the right and the given information.

Given: $\overline{LP} \parallel \overline{NO}, \overline{LP} \cong \overline{NO}$



20. From $\overline{LP} \parallel \overline{NO}$, which angles are congruent and why?
21. From looking at the picture, what additional piece of information can you conclude?
22. Write a 2-column proof to prove $\triangle LMP \cong \triangle OMN$.
23. What would be your reason for $\overline{LM} \cong \overline{MO}$?
24. Fill in the blanks for the proof below. Use the given and the picture from above. Prove: M is the midpoint of $\overline{PN}$

TABLE 4.16:

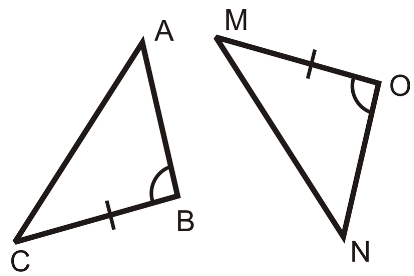
Statement	***Reason***
1. $\overline{LP} \parallel \overline{NO}, \overline{LP} \cong \overline{NO}$	Given

TABLE 4.16: (continued)

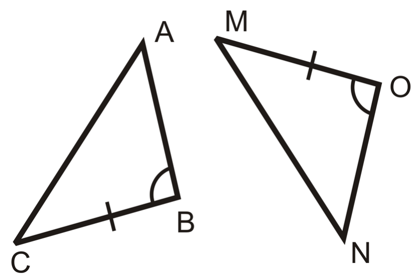
Statement	***Reason***
2.	Alternate Interior Angles
3.	ASA
4. $\overline{LM} \cong \overline{MO}$	
5. M is the midpoint of $\overline{PN}$	

Determine the additional piece of information needed to show the two triangles are congruent by the given postulate.

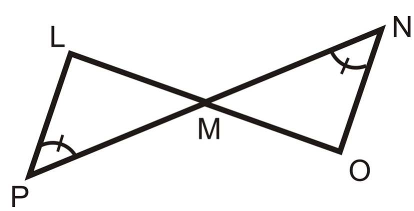
25. AAS



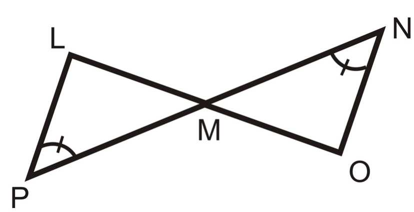
26. ASA



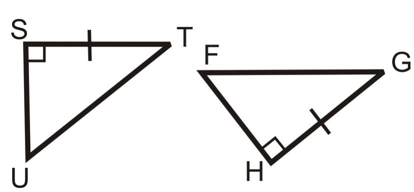
27. ASA



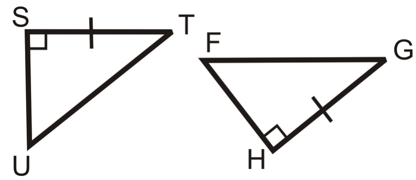
28. AAS



29. HL

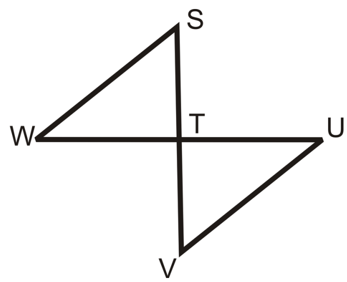


30. SAS

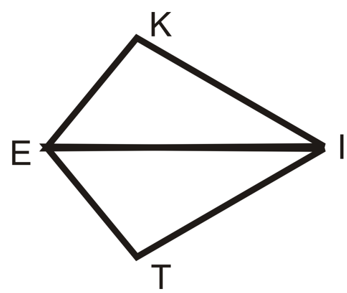


Write a 2-column proof.

31. Given: $\overline{SV} \perp \overline{WU} T$ is the midpoint of $\overline{SV}$ and $\overline{WU}$ Prove: $\overline{WS} \cong \overline{UV}$



32. Given: $\angle K \cong \angle T$, $\overline{EI}$ is the angle bisector of $\angle KET$ Prove: $\overline{EI}$ is the angle bisector of $\angle KIT$



Review Queue Answers

1.

TABLE 4.17:

Statement	***Reason***
1. $\overline{AD} \cong \overline{DC}$, $\overline{AB} \cong \overline{CB}$	Given
2. $\overline{DB} \cong \overline{DB}$	Reflexive PoC
3. $\triangle DAB \cong \triangle DCB$	SSS

2. No, only the angles are congruent, you need at least one side to prove the triangles are congruent.

3. (a) Yes, CPCTC

(b) No, these sides do not line up in the congruence statement.

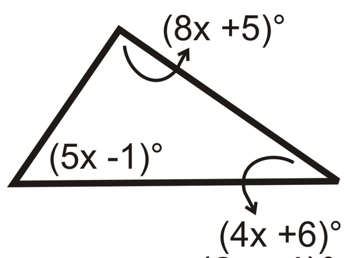
4.5 Isosceles and Equilateral Triangles

Learning Objectives

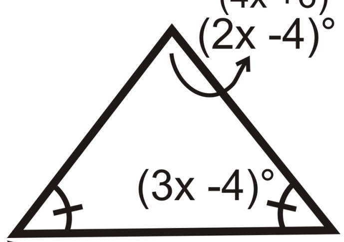
- Understand the properties of isosceles and equilateral triangles.
- Use the Base Angles Theorem and its converse.
- Prove an equilateral triangle is also equiangular.

Review Queue

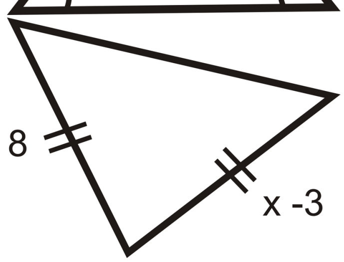
Find the value of x.



a.



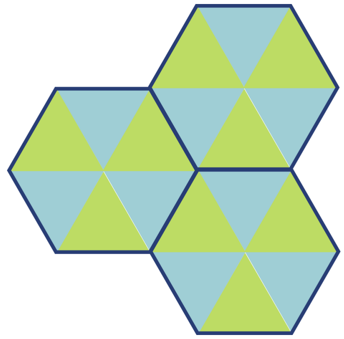
b.



c.

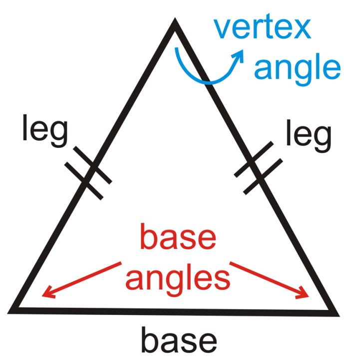
d. If a triangle is equiangular, what is the measure of each angle?

Know What? Your parents now want to redo the bathroom. To the right is the tile they would like to place in the shower. The blue and green triangles are all equilateral. What type of polygon is dark blue outlined figure? Can you determine how many degrees are in each of these figures? Can you determine how many degrees are around a point? HINT: For a "point" you can use a point where the six triangles meet.



Isosceles Triangle Properties

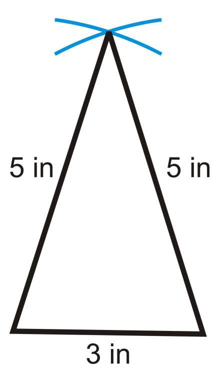
An isosceles triangle is a triangle that has ***at least*** two congruent sides. The congruent sides of the isosceles triangle are called the ***legs***. The other side is called the **base** and the angles between the base and the congruent sides are called **base angles**. The angle made by the two legs of the isosceles triangle is called the **vertex angle.**



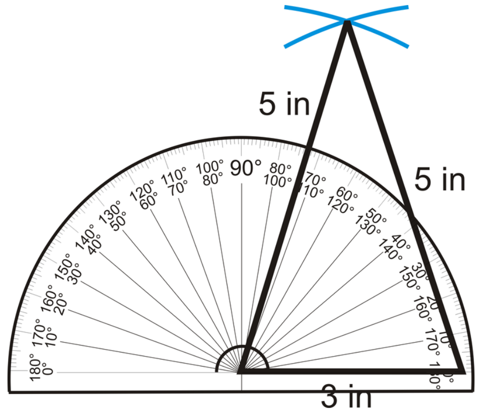
Investigation 4-5: Isosceles Triangle Construction

Tools Needed: pencil, paper, compass, ruler, protractor

a. Refer back to Investigation 4-2. Using your compass and ruler, draw an isosceles triangle with sides of 3 in, 5 in and 5 in. Draw the 3 in side (the base) horizontally 6 inches from the top of the page.



b. Now that you have an isosceles triangle, use your protractor to measure the base angles and the vertex angle.



The base angles should each be 72.5° *and the vertex angle should be* 35°.

We can generalize this investigation into the Base Angles Theorem.

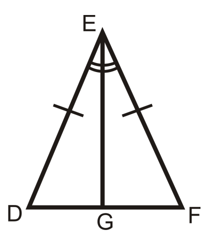
Base Angles Theorem: The base angles of an isosceles triangle are congruent.

To prove the Base Angles Theorem, we will construct the angle bisector (Investigation 1-5) through the vertex angle of an isosceles triangle.

Given: Isosceles triangle $\triangle DEF$ with $\overline{DE} \cong \overline{EF}$

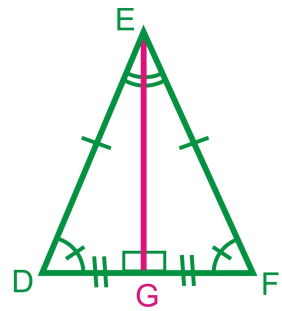
Prove: $\angle D \cong \angle F$

TABLE 4.18:

Statement	***Reason***
1. Isosceles triangle $\triangle DEF$ with $\overline{DE} \cong \overline{EF}$	Given
2. Construct angle bisector $\overline{EG}$ for $\angle E$ 	Every angle has one angle bisector
3. $\angle DEG \cong \angle FEG$	Definition of an angle bisector
4. $\overline{EG} \cong \overline{EG}$	Reflexive PoC
5. $\triangle DEG \cong \triangle FEG$	SAS
6. $\angle D \cong \angle F$	CPCTC

By constructing the angle bisector, $\overline{EG}$, we designed two congruent triangles and then used CPCTC to show that the base angles are congruent. Now that we have proven the Base Angles Theorem, you do not have to construct the angle bisector every time. It can now be assumed that base angles of any isosceles triangle are always equal.

Let's further analyze the picture from step 2 of our proof.



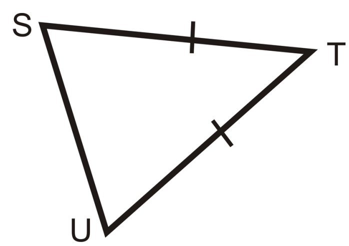
Because $\triangle DEG \cong \triangle FEG$, we know that $\angle EGD \cong \angle EGF$ by CPCTC. Thes two angles are also a linear pair, so they are congruent supplements, or 90° each. Therefore, $\overline{EG} \perp \overline{DF}$.

Additionally, $\overline{DG} \cong \overline{GF}$ by CPCTC, so G is the midpoint of $\overline{DF}$. This means that $\overline{EG}$ is the **perpendicular bisector** of $\overline{DF}$, in addition to being the angle bisector of $\angle DEF$.

Isosceles Triangle Theorem: The angle bisector of the vertex angle in an isosceles triangle is also the perpendicular bisector to the base.

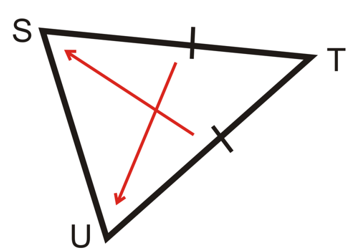
This is ONLY true for the vertex angle. We will prove this theorem in the review questions for this section.

Example 1: Which two angles are congruent?



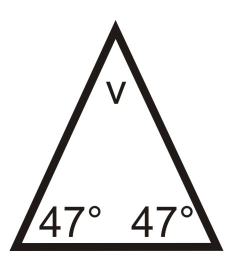
Solution: This is an isosceles triangle. The congruent angles, are opposite the congruent sides.

From the arrows we see that $\angle S \cong \angle U$.



Example 2: If an isosceles triangle has base angles with measures of 47°, what is the measure of the vertex angle?

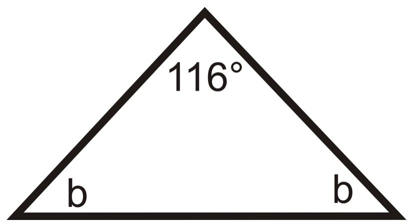
Solution: Draw a picture and set up an equation to solve for the vertex angle, v.



$$47^\circ + 47^\circ + v = 180^\circ$$
$$v = 180^\circ - 47^\circ - 47^\circ$$
$$v = 86^\circ$$

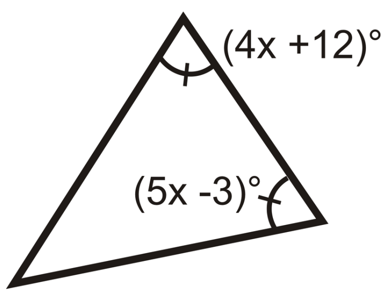
Example 3: If an isosceles triangle has a vertex angle with a measure of 116°, what is the measure of each base angle?

Solution: Draw a picture and set up and equation to solve for the base angles, b. Recall that the base angles are equal.



$$116^\circ + b + b = 180^\circ$$
$$2b = 64^\circ$$
$$b = 32^\circ$$

Example 4: ***Algebra Connection*** Find the value of x and the measure of each angle.



Solution: Set the angles equal to each other and solve for x.

$$(4x + 12)^\circ = (5x - 3)^\circ$$
$$15^\circ = x$$

If $x = 15^\circ$, then the base angles are $4(15^\circ) + 12^\circ$, or 72°. The vertex angle is $180^\circ - 72^\circ - 72^\circ = 36^\circ$.

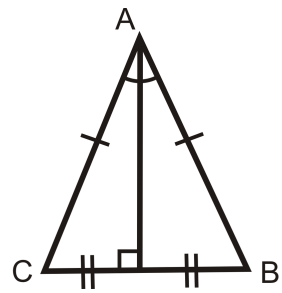
The converses of the Base Angles Theorem and the Isosceles Triangle Theorem are both true.

Base Angles Theorem Converse: If two angles in a triangle are congruent, then the opposite sides are also congruent.

So, for a triangle $\triangle ABC$, if $\angle A \cong \angle B$, then $\overline{CB} \cong \overline{CA}$. $\angle C$ would be the vertex angle.

Isosceles Triangle Theorem Converse: The perpendicular bisector of the base of an isosceles triangle is also the angle bisector of the vertex angle.

In other words, if $\triangle ABC$ is isosceles, $\overline{AD} \perp \overline{CB}$ and $\overline{CD} \cong \overline{DB}$, then $\angle CAD \cong \angle BAD$.



Equilateral Triangles

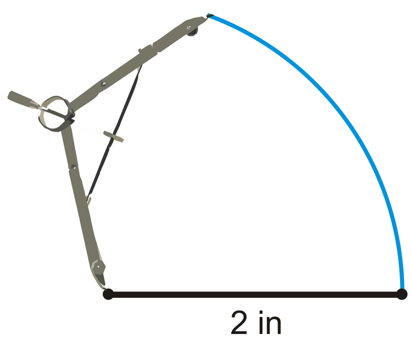
By definition, all sides in an equilateral triangle have exactly the same length. Therefore, ***every equilateral triangle is also an isosceles triangle.***

Investigation 4-6: Constructing an Equilateral Triangle

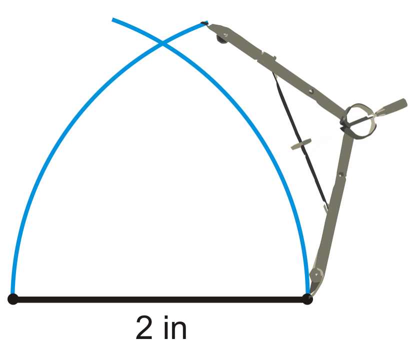
Tools Needed: pencil, paper, compass, ruler, protractor

1. Because all the sides of an equilateral triangle are equal, pick a length to be all the sides of the triangle. Measure this length and draw it horizontally on your paper.

2. Put the pointer of your compass on the left endpoint of the line you drew in Step 1. Open the compass to be the same width as this line. Make an arc above the line.

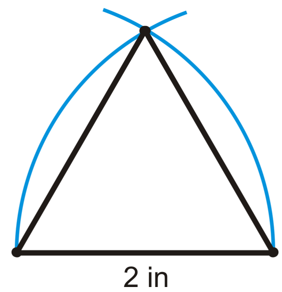


3. Repeat Step 2 on the right endpoint.



4. Connect each endpoint with the arc intersections to make the equilateral triangle.

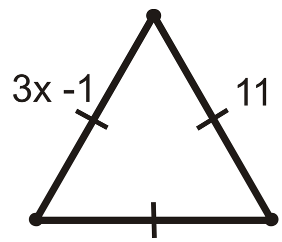
Use the protractor to measure each angle of your constructed equilateral triangle. What do you notice?



From the Base Angles Theorem, the angles opposite congruent sides in an isosceles triangle are congruent. So, if all three sides of the triangle are congruent, then all of the angles are congruent or 60° each.

Equilateral Triangles Theorem: All equilateral triangles are also equiangular. Also, all equiangular triangles are also equilateral.

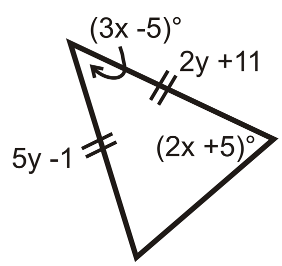
Example 5: ***Algebra Connection*** Find the value of x.



Solution: Because this is an equilateral triangle $3x-1=11$. Now, we have an equation, solve for x.

$$\begin{aligned}3x-1&=11\\3x&=12\\x&=4\end{aligned}$$

Example 6: ***Algebra Connection*** Find the values of x and y.



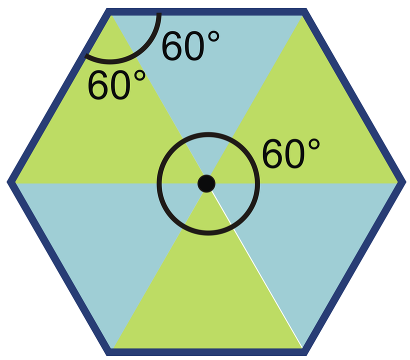
Solution: Let's start with y. Both sides are equal, so set the two expressions equal to each other and solve for y.

$$\begin{aligned}5y-1&=2y+11\\3y&=12\\y&=4\end{aligned}$$

For x, we need to use two $(2x+5)^\circ$ expressions because this is an isosceles triangle and that is the base angle measurement. Set all the angles equal to 180° and solve.

$$\begin{aligned}(2x+5)^\circ + (2x+5)^\circ + (3x-5)^\circ &= 180^\circ \\ (7x+5)^\circ &= 180^\circ \\ 7x &= 175^\circ \\ x &= 25^\circ\end{aligned}$$

Know What? Revisited Let's focus on one tile. First, these triangles are all equilateral, so this is an equilateral hexagon (6 sided polygon). Second, we now know that every equilateral triangle is also equiangular, so every triangle within this tile has 360° angles. This makes our equilateral hexagon also equiangular, with each angle measuring 120°. Because there are 6 angles, the sum of the angles in a hexagon are 6.120° or 720°. Finally, the point in the center of this tile, has 660° angles around it. That means there are 360° around a point.

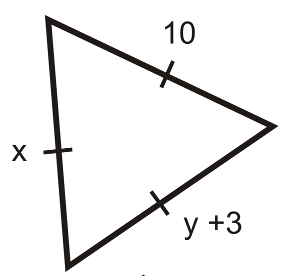


Review Questions

Constructions For questions 1-5, use your compass and ruler to:

1. Draw an isosceles triangle with sides 3.5 in, 3.5 in, and 6 in.
2. Draw an isosceles triangle that has a vertex angle of 100° and legs with length of 4 cm. (you will also need your protractor for this one)
3. Draw an equilateral triangle with sides of length 7 cm.
4. Using what you know about constructing an equilateral triangle, construct (without your protractor) a 60° angle.
5. Draw an isosceles right triangle. What is the measure of the base angles?

For questions 6-17, find the measure of x and/or y.



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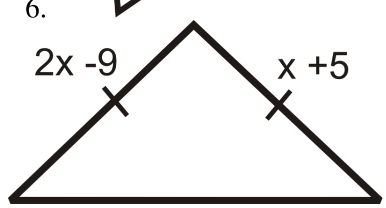
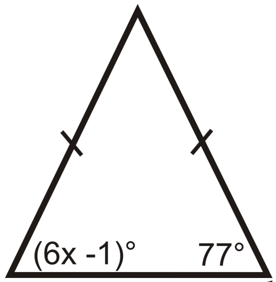


7.



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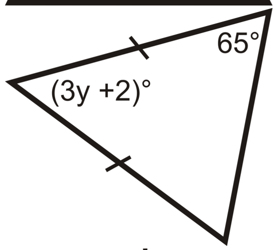

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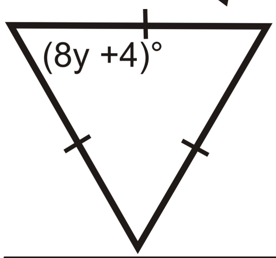

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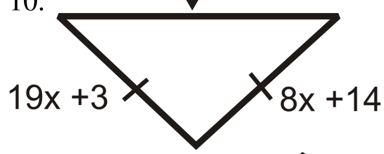

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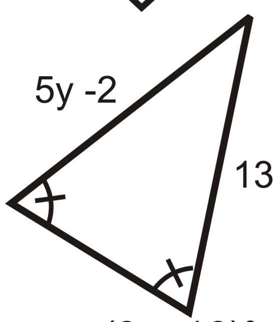

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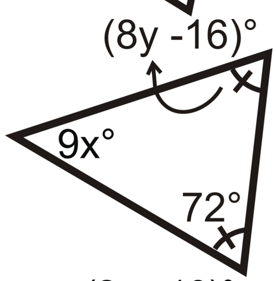

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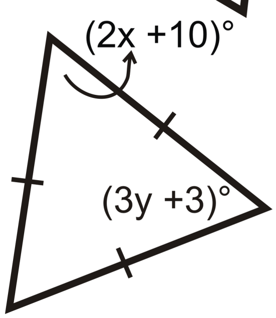
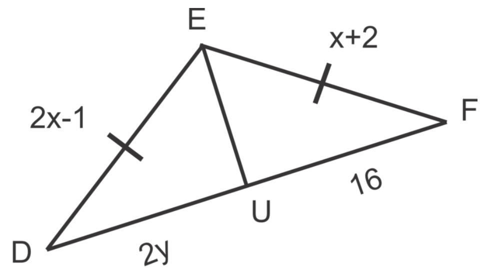
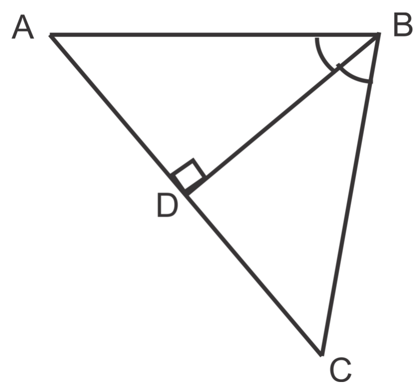


14.

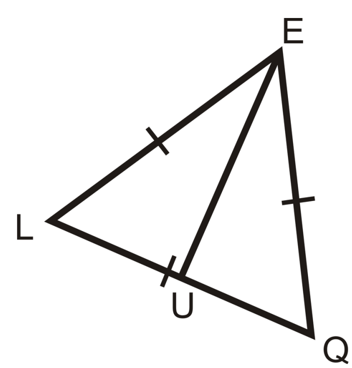
15. $\angle DEF$ in triangle $\triangle DEF$ is bisected by $\overline{EU}$. Find x and y.



16. Is $\triangle ABC$ isosceles? Explain your reasoning.



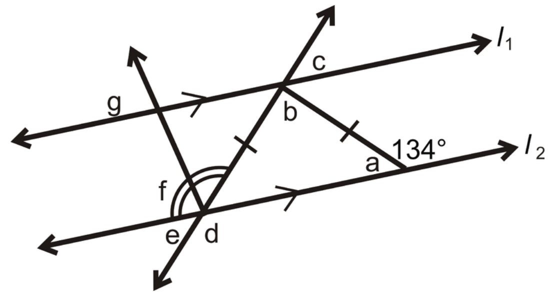
17. $\triangle EQG$ is an equilateral triangle. If $\overline{EU}$ bisects $\angle LEQ$, find:



a. $m\angle EUL$
b. $m\angle UEL$
c. $m\angle ELQ$
d. If $EQ = 4$, find LU.

Determine if the following statements are ALWAYS, SOMETIMES, or NEVER true. Explain your reasoning.

18. Base angles of an isosceles triangle are congruent.
19. Base angles of an isosceles triangle are complementary.
20. Base angles of an isosceles triangle can be equal to the vertex angle.
21. Base angles of an isosceles triangle can be right angles.
22. Base angles of an isosceles triangle are acute.
23. In the diagram below, $l_1 \parallel l_2$. Find all of the lettered angles.



Fill in the blanks in the proofs below.

24. Given: Isosceles $\triangle CIS$, with base angles $\angle C$ and $\angle S\overline{IO}$ is the angle bisector of $\angle CIS$Prove: $\overline{IO}$ is the perpendicular bisector of $\overline{CS}$

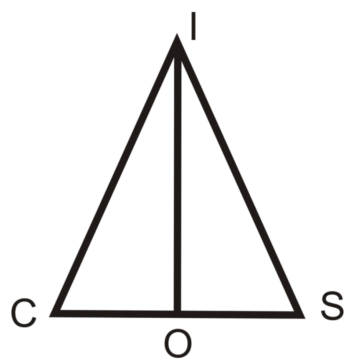
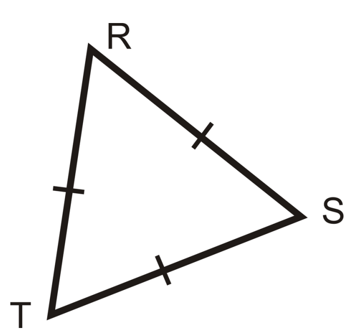


TABLE 4.19:

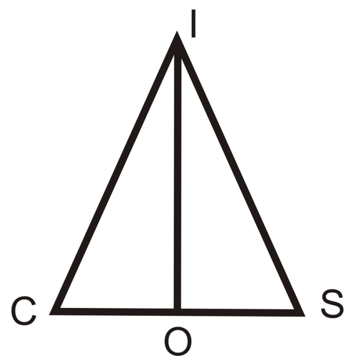
Statement	*Reason*
1.	Given
2.	Base Angles Theorem
3. $\angle CIO \cong \angle SIO$	
4.	Reflexive PoC
5. $\triangle CIO \cong \triangle SIO$	
6. $\overline{CO} \cong \overline{OS}$	
7.	CPCTC
8. $\angle IOC$ and $\angle IOS$ are supplementary	
9.	Congruent Supplements Theorem
10. $\overline{IO}$ is the perpendicular bisector of $\overline{CS}$	

Write a 2-column proof.

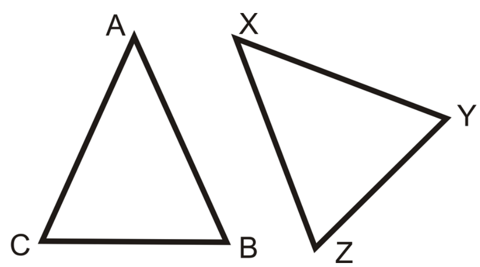
25. Given: Equilateral $\triangle RST$ with $\overline{RT} \cong \overline{ST} \cong \overline{RS}$Prove: $\triangle RST$ is equiangular



26. Given: Isosceles $\triangle ICS$ with $\angle C$ and $\angle S\overline{IO}$ is the perpendicular bisector of $\overline{CS}$Prove: $\overline{IO}$ is the angle bisector of $\angle CIS$



27. Given: Isosceles $\triangle ABC$ with base angles $\angle B$ and $\angle C$ Isosceles $\triangle XYZ$ with base angles $\angle Y$ and $\angle Z\angle C \cong \angle Z, \overline{BC} \cong \overline{YZ}$Prove: $\triangle ABC \cong \triangle XYZ$



Constructions

28. Using the construction of an equilateral triangle (investigation 4-6), construct a 30° angle. *Hint: recall how to bisect an angle from investigation 1-4.*
29. Use the construction of a 60° angle to construct a 120° angle.
30. Is there another way to construction a 120° angle? Describe the method.
31. Describe how you could construct a 45° angle (there is more than one possible way).

Review Queue Answers

a. $$\begin{aligned}(5x-1)^\circ + (8x+5)^\circ + (4x+6)^\circ &= 180^\circ \\ 17x+10 &= 180^\circ \\ 17x &= 170^\circ \\ x &= 10^\circ\end{aligned}$$

b. $$\begin{aligned}(2x-4)^\circ + (3x-4)^\circ + (3x-4)^\circ &= 180^\circ \\ 8x-12 &= 180^\circ \\ 8x &= 192^\circ \\ x &= 24^\circ\end{aligned}$$

c. $$\begin{aligned}x-3 &= 8 \\ x &= 5\end{aligned}$$

d. Each angle is $\frac{180^\circ}{3}$, or 60°

4.6 Chapter 4 Review

Definitions, Postulates, and Theorems

Interior Angles
The angles inside of a closed figure with straight sides.

Vertex
The point where the sides of a polygon meet.

Triangle Sum Theorem
The interior angles of a triangle add up to 180°.

Exterior Angle
The angle formed by one side of a polygon and the extension of the adjacent side.

Exterior Angle Sum Theorem
Each set of exterior angles of a polygon add up to 360°.

Remote Interior Angles
The two angles in a triangle that are not adjacent to the indicated exterior angle.

Exterior Angle Theorem
The sum of the remote interior angles is equal to the non-adjacent exterior angle.

Congruent Triangles
Two triangles are congruent if the three corresponding angles and sides are congruent.

Third Angle Theorem
If two angles in one triangle are congruent to two angles in another triangle, then the third pair of angles must also congruent.

Reflexive Property of Congruence
Any shape is congruent to itself.

Symmetric Property of Congruence
If two shapes are congruent, the statement can be written with either shape on either side of the $\cong$ sign.

Transitive Property of Congruence
If two shapes are congruent and one of those is congruent to a third, the first and third shapes are also congruent.

Side-Side-Side (SSS) Triangle Congruence Postulate
If three sides in one triangle are congruent to three sides in another triangle, then the triangles are congruent.

Included Angle
When an angle is between two given sides of a triangle (or polygon).

Side-Angle-Side (SAS) Triangle Congruence Postulate
If two sides and the included angle in one triangle are congruent to two sides and the included angle in another triangle, then the two triangles are congruent.

Angle-Side-Angle (ASA) Congruence Postulate
If two angles and the included side in one triangle are congruent to two angles and the included side in another triangle, then the two triangles are congruent.

Angle-Angle-Side (AAS or SAA) Congruence Theorem
If two angles and a non-included side in one triangle are congruent to two corresponding angles and a non-included side in another triangle, then the triangles are congruent.

HL Congruence Theorem
If the hypotenuse and leg in one right triangle are congruent to the hypotenuse and leg in another right triangle, then the two triangles are congruent.

Base Angles Theorem
The base angles of an isosceles triangle are congruent.

Isosceles Triangle Theorem
The angle bisector of the vertex angle in an isosceles triangle is also the perpendicular bisector to the base.

Base Angles Theorem Converse
If two angles in a triangle are congruent, then the opposite sides are also congruent.

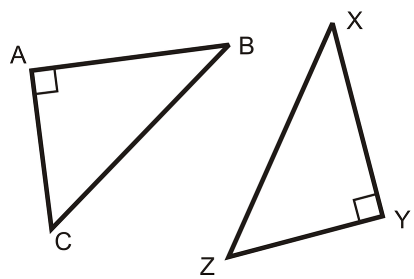
Isosceles Triangle Theorem Converse
The perpendicular bisector of the base of an isosceles triangle is also the angle bisector of the vertex angle.

Equilateral Triangles Theorem
all sides in an equilateral triangle have exactly the same length.

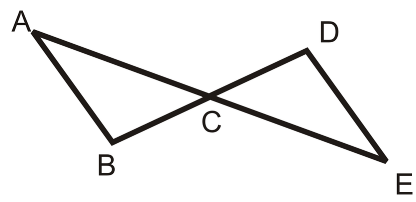
Review

For each pair of triangles, write what needs to be congruent in order for the triangles to be congruent. Then, write the congruence statement for the triangles.

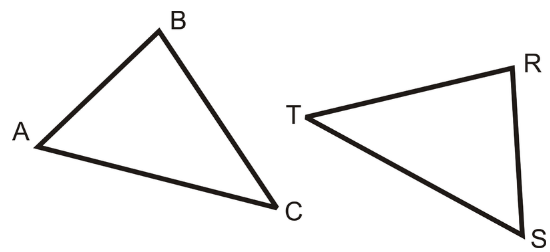
1. HL



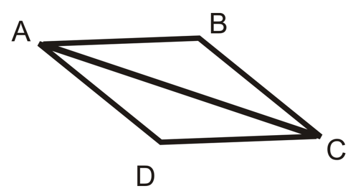
2. ASA



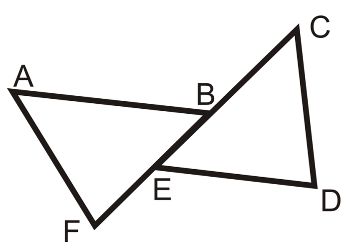
3. AAS



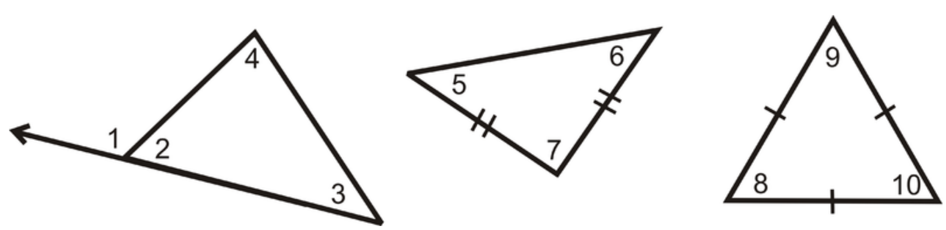
4. SSS



5. SAS



Using the pictures below, determine which theorem, postulate or definition that supports each statement below.



6. $m\angle 1 + m\angle 2 = 180^\circ$
7. $\angle 5 \cong \angle 6$
8. $m\angle 1 = m\angle 4 + m\angle 3$
9. $m\angle 8 = 60^\circ$
10. $m\angle 5 + m\angle 6 + m\angle 7 = 180^\circ$
11. $\angle 8 \cong \angle 9 \cong \angle 10$
12. If $m\angle 7 = 90^\circ$, then $m\angle 5 = m\angle 6 = 45^\circ$

Texas Instruments Resources

In the CK-12 Texas Instruments Geometry FlexBook, there are graphing calculator activities designed to supplement the objectives for some of the lessons in this chapter. See http://www.ck12.org/flexr/chapter/9689 .